Fluids and Pressure

Density ρ = mass/volume [kg/m^3]

-Specific gravity SG = $\rho_{object}/\rho_{water}$ If SG>1 sinks, if SG <1 floats
- Specific gravity SG = fraction of floating object below the water
- ρ_{water} = 1000 kg/m^3

Pressure P = Force/Area [N/m^2 = Pascals]

- $P_{atm} = 1.01 \times 10^5$ Pa
- A small force can exert a huge pressure if area of contact is small (e.g. pushing tack into wall)

$P_{absolute}=P_o + \rho gh$ **$P_{gauge}= \rho gh$**

- P_o is the pressure above the fluid, which is usually $P_{atm} = 1.01 \times 10^5$ Pa
- P_{gauge} measures the difference between the fluid pressure and atmosphereic pressure
- P_{gauge} is what you measure when you check your tires

Volume flow rate = (Area)(speed) [m^3/sec]

$A_1v_1=A_2v_2$ (or **Av = constant**)

- Note: This is true for liquids, not gases, because liquids are incompressible

Buoyancy

- **Archimedes principle**: The Buoyant force equals the weight of the fluid displaced

$F_{buoyancy} = \rho_{fluid} V_{fluid\ displaced}\, g$

- The $V_{fluid\ displaced}$ is not always the volume of the object! (unless it is completely submerged)
- $F_{buoyancy}$ = Weight in air – Weight in water (for objects that don't float)

Bernoulli's Equation

$$P_1 + ½\rho v_1^2 + \rho gh_1 = P_2 + ½\rho v_2^2 + \rho gh_2$$

Note: If the height does not change significantly, Bernoulli's eqn says,

$$P_1 + ½\rho v_1^2 = P_2 + ½\rho v_2^2$$ (or **$P + ½\rho v^2$= constant**)

- **Bernoulli's principle**: If the speed of a fluid increases, the pressure of the fluid decreases
- e.g. Blowing air over paper decreases pressure on top, and the paper rises

Thermal Physics

Thermal expansion $\Delta L = \alpha L_o \Delta T$ [m]

- Note: ΔT can be in ^oC or K, but if you have an equation with just **T** it must be in K
- Almost all objects expand when heated (note: holes in heated objects also expand)

Thermal conduction $Q/time = kA\,\Delta T/L$ [J/sec or Watts]

- Heat Q passes through a material of area A, thickness L, and thermal conductivity k
- **Conduction** is when heat flows through an object, **convection** is when currents of hot fluid flow to colder regions, and **radiation** is when energy is transferred by EM waves

$PV = nRT$ (n=moles, T in K, R = 8.31 Pa m^3 /mol K, R = 0.082 atm L /mol K)

$PV = Nk_BT$ (N = # molecules, T in K, P in Pa, V in m^3, $k_B = 1.38 \times 10^{-23}$ J/K)

$Q_{gained} = Q_{lost}$ (note: Heat is measured in Joules)

$Q = mc\,\Delta T$ (use when object is changing temperature)

$Q = mL$ (use when object is changing phase, e.g. solid to liquid or liquid to gas)

- use **heat of fusion** for (solid <-> liquid) use **heat of vaporization** for (liquid <-> gas)

$V_{rms} = [3k_BT/M_{molecule}]^{1/2}$ (root-mean-square speed of molecule in an ideal gas)

Laws of thermodynamics

First Law: $\Delta U = Q_{\text{heat flows into gas}} + W_{\text{work done on gas}}$

- Internal energy (U) increases if heat Q flows into a gas, or work W is done on a gas
- If internal energy of a gas increases, then T increases.
- For Monatomic Ideal Gas you can use the equation $\Delta U = 3/2\,\Delta(PV)$

Second Law: Entropy of universe is always increasing

- heat never flows spontaneously from cold to hot
- no heat engine can have an efficiency of 1

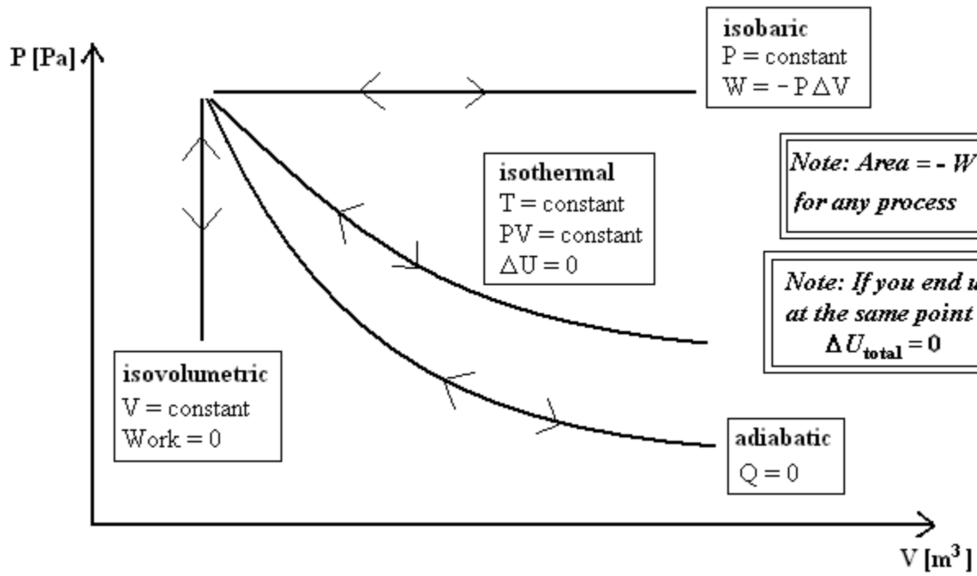
$\text{efficiency} = W/Q_H = 1 - Q_C/Q_H$ (Q_c is waste heat created by heat engine)

- The most efficient engine (Carnot engine) has efficiency of **$e = 1 - T_C/T_H$**

Change in entropy $\Delta S = Q/T$

- if heat flows into an object, that object's entropy increases (and vice versa)

Common thermal processes



Important things to know about PV diagrams

moving to right is expansion (- work is done on gas)

moving to left is compression (+ work is done on gas)

Work done <u>on</u> gas = - Work done <u>by</u> gas

Moving to larger PV (up and right) means ΔU is + (and since W is -, Q must be +)

Moving to lower PV (down and left) means ΔU is - (and since W is +, Q must be -)

Example 1

An ice cube at -35C is dropped into a 100g aluminum container that holds 300g of water at 50C. If the equilibirum temperature reached is 20C, what was the mass of the ice cube?

c_{water} = 4186 J/kg C

c_{ice} = 2100 J/kg C

$c_{aluminum}$ = 900 J/kg C

L_{fusion} = 330,000 J/kg

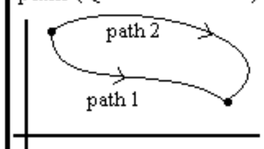
$Q_{gained} = Q_{lost}$ *(Note: ice cube gains heat, but water and aluminum lose heat)*

m(2100)[0C - (-35C)] + m(330,000) +m(4186)(20C-0) = (0.1kg)(900)(50C-20C) + (0.3kg)(4186)(50C-20C)

m(487220) = 40374

m = 0.083 kg

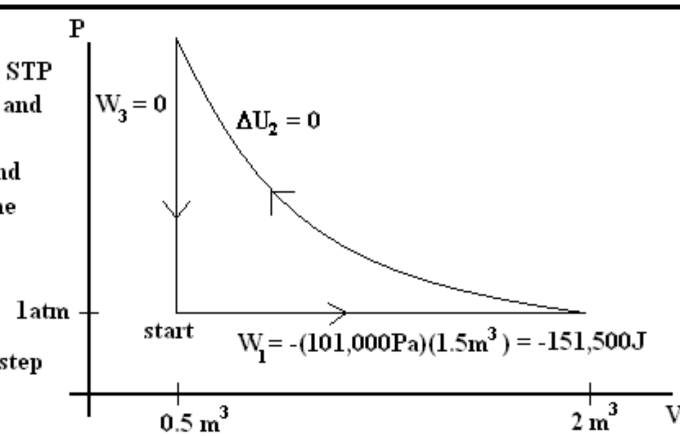
If two paths start at the same point, and end up at the same point, ΔU will be the same for both paths (Q and W will not)



Example 2

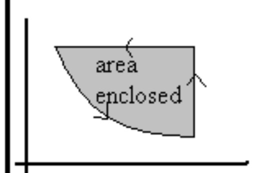
A 0.5 m^3 sample of an ideal gas starts at STP and expands to 2 m^3 at constant pressure and absorbs 200,000J of heat. Then the gas is compressed isothermally back to 0.5 m^3 and the gas does 320,000J of work. Finally the gas is cooled at constant volume back to 1atm.

Draw the process on a PV diagram and determine the ΔU, Q and W for each step and the whole process.



path 1	path 2	path 3	total path
W=-151,500 J	W= 320,000J	W=0	W=168,500J
Q = +200,000J	Q = -320,000	Q= -48,500J	Q = -168,500J
ΔU= +48,500J	ΔU = 0	ΔU= -48,500J	ΔU = 0

The total area enclosed by a closed path will tell you the total work done (could be + or -)



For an enclosed area the total work done on gas is + if top line goes to the left (as seen in the above diagram)

Electric Forces, Fields, Energy & Voltage

F_e is **electric force** [Newtons]

+ charges feel force in same dir of the electric field (E), - charges feel force in opp. dir of electric field (E)

E is **electric field** [N/Coulomb or Volts/meter]

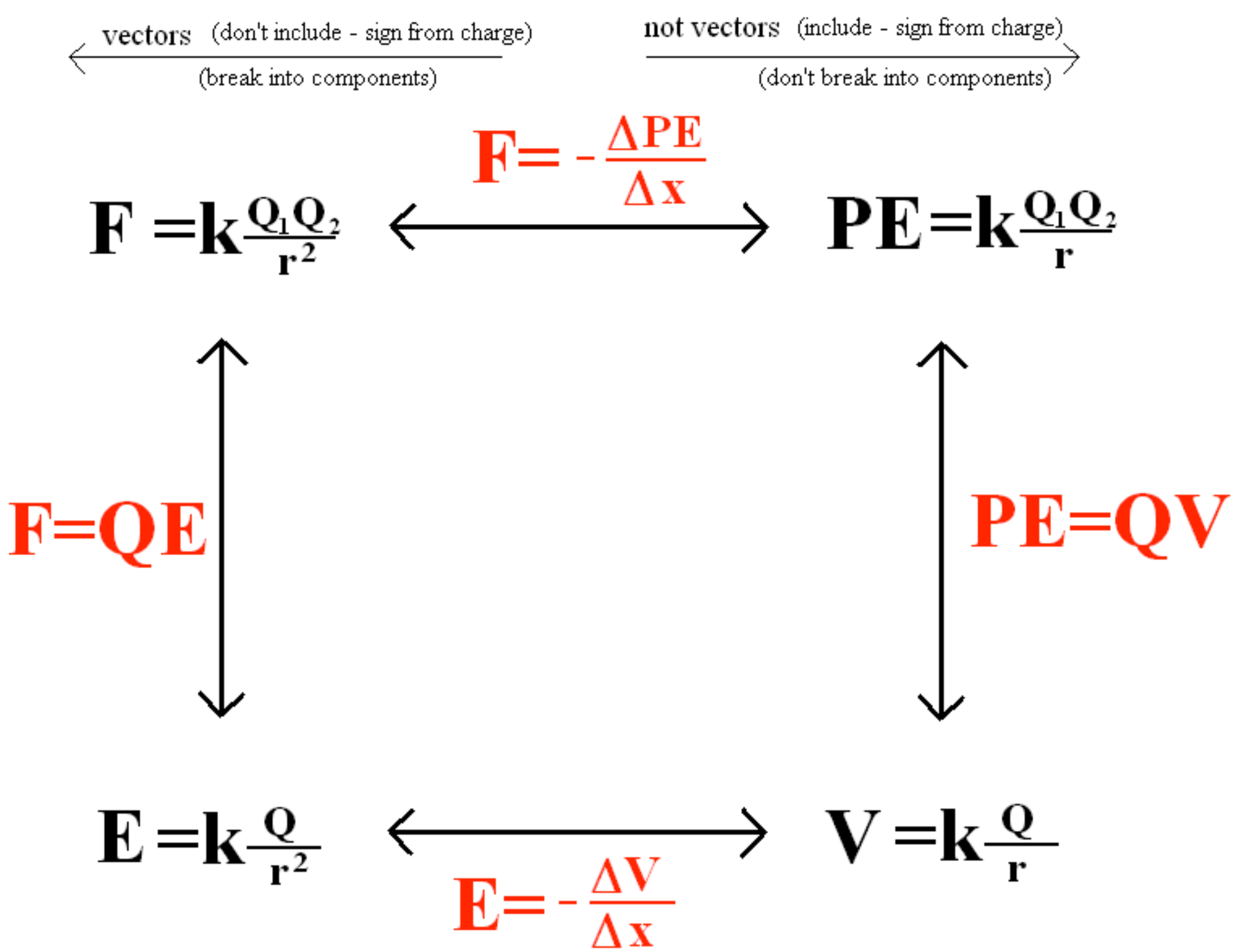
+ charges create electric fields that point radially outward from charge, - charges create E pointing inward

$PE_{electric}$ is **electric potential energy** [Joules]

Electric Poetential Energy (PE) is another form of energy that objects can have

V is **electric potential** [J/Coulomb or Volts]

Electric Potential (V) at a point is the Electric Potential Energy (PE) 1C of charge would have at that point

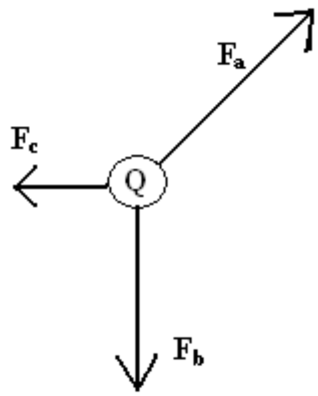


Note: + and - charges both feel a force **toward lower $PE_{electric}$** , also **Electric fields E** point toward **lower V**

Solving 2D electric force/field problems

1. Draw the forces <u>exerted on</u> the charge you are concerned with

(These are forces <u>ON</u> charge Q, not the forces charge Q is exerting on other charges)



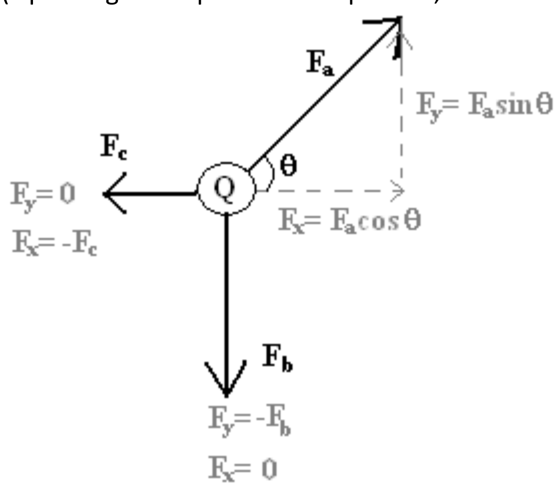
2. Find the size of each force using $\mathbf{F=kq_1q_2/r^2}$

3. Break forces into vertical and horizontal **components F_y and F_x**

(For completely vertical or horizontal forces one component will be zero, the other is $\pm$ F)

(For diagonal forces you need to use sin and cos to break the vector into components)

(Up or Right is a positive component, Left or Down is a negative component)



4. Add up all the **F_x and F_y** to get the components of the total Force vector F_{xtot} and F_{ytot}

i.e. for the example shown $F_{xtot} = F_a \cos\theta + 0 + (-F_c)$ $F_{ytot} = F_a \sin\theta + (-F_b) + 0$

5. Find magnitude of total Force using the **Pythagorean Theorem**

i.e. $F_{tot}^2 = F_{xtot}^2 + F_{ytot}^2$

6. Find the **angle** from horizontal by using θ **= $\tan^{-1}$ (F_{ytot}/F_{xtot})**

(Note: This angle is always the angle <u>from the "nearest" x-axis</u> to the total Force vector)

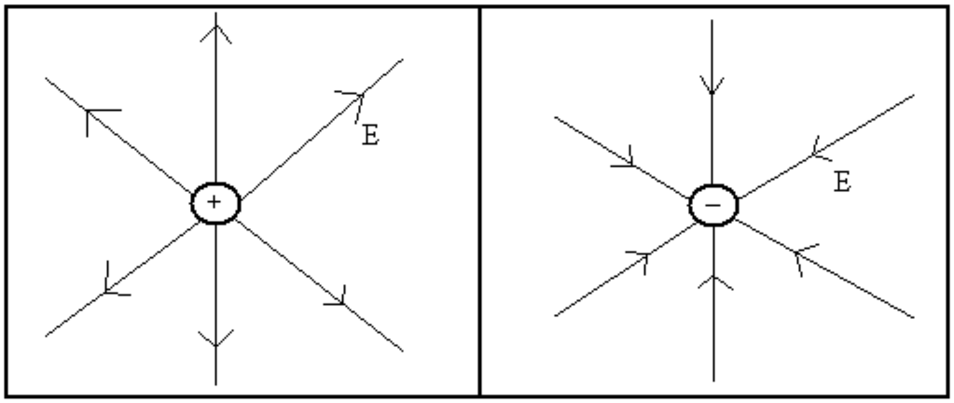
Electric Field Lines and Equipotential Lines

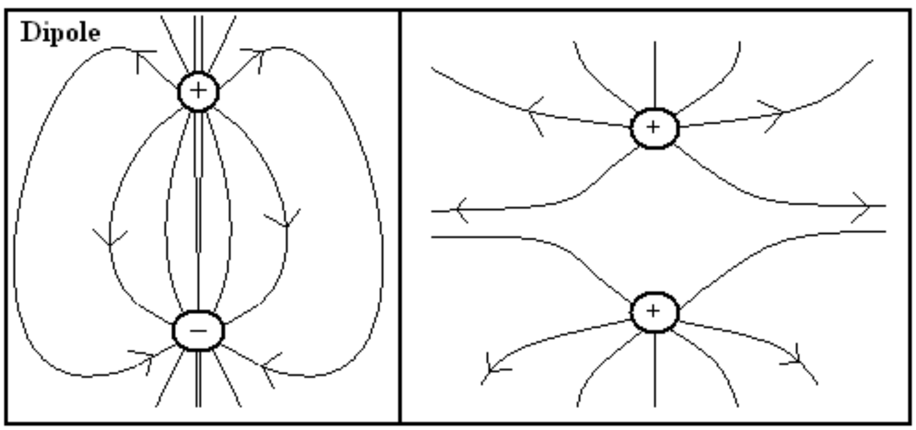
Electric Field Lines show the direction of the electric field (E) at points in space.

Field lines point radially outward from "+" charges, and radially inward toward "-" charges.

The Electric field E is greatest where field lines are closest together.

Larger charges get more field lines to start/end on them.



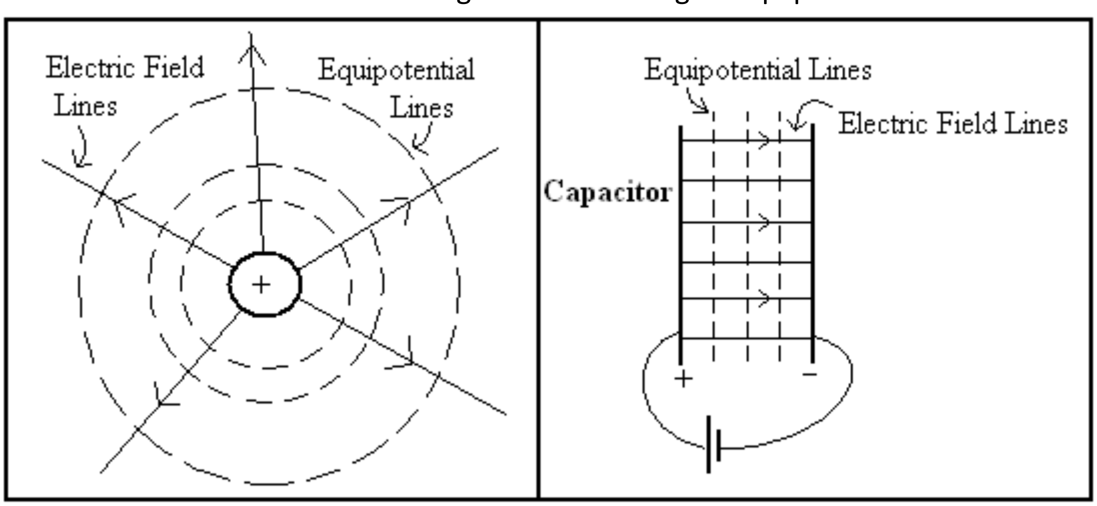


Equipotential Lines show lines of constant electric potential (V).

Equipotential Lines are always perpendicular to the Electric Field Lines.

The Electric Field E is greatest where Equipotential Lines are closest together, $\mathbf{E=-\Delta V/\Delta x}$

There is no work done when a charge is moved along an Equipotential Line.



Circuits

Current

I = Q/t [C/sec = Amperes]

- defined to be in the direction of positive charge flow (or opposite direction of e^-)
- is directed out of the + terminal of a battery, and into the - terminal

Resistance

The resistance of a length **L** of cylinder made with resistivity **ρ**, and cross sectional area **A** is,

R=ρ L/A [Ohms]

Ohm's Law

V = IR (V is voltage drop across resistor, I is current through the resistor, R is resistance)

- V is not necessarily the voltage of the battery!
- **Ohmic materials** have constant resistance (slope on V vs. I), regardless of what the current is
- **Non-Ohmic materials** change their "resistance" depending on what the current/voltage is

Electrical Power

P = IV [Watts]

$$P = I^2R$$

$$P = V^2/R$$

Capacitors

C = Q/V (C is capacitance, Q is charge on + plate, V is voltage across capacitor)

- Capacitance tells you how well a capacitor can store charge
- Inserting a **Dielectric** between a capacitor always **increases capacitance** by a factor of **k**
- Capacitors store energy as well, which is given by

$$E_{capacitor} = \frac{1}{2} QV = \frac{1}{2} CV^2$$

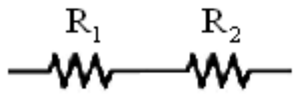
- For a parallel plate capacitor with plates of area A separated by a distance d, capacitance is,

$$C = \varepsilon_o A/d$$

Combining Resistors

Resistors in Series

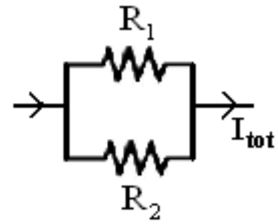
$$R_{eq} = R_1 + R_2$$



Note: Resistors in Series always have same current

Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



Note: Resistors in Parallel always have same voltage

If circuit has only 1 battery,

Choose resistors, two at a time, and reduce to a single resistor to determine the current through the battery. Then determine how current breaks up at junctions using these rules ↗

If one resistor is 3 times larger than the other, smaller resistor gets 3/4 of the total current $I_{R_1} = \frac{3}{4} I_{tot}$

If one resistor is 5 times larger than the other, smaller resistor gets 5/6 the total current $I_{R_1} = \frac{5}{6} I_{tot}$

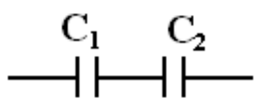
or, if resistors are not a nice ratio use this formula

$$I_{R_1} = I_{tot} \frac{R_2}{(R_1 + R_2)}$$

Combining Capacitors

Capacitors in Series

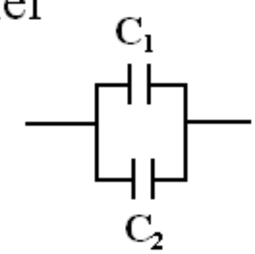
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



Note: Capacitors in series all have same charge

Capacitors in Parallel

$$C_{eq} = C_1 + C_2$$



Note: Capacitors in parallel all have same voltage

When you reduce all capacitors to a single C, you can find Q (=CV). Then work backwards to find Q on each capacitor.

For capacitors in parallel, if one capacitor has **3** times more capacitance than the other, it gets **3/4** of the total charge.

Or, if not a nice ratio use,

$$Q_1 = Q_{tot} \frac{C_1}{C_1 + C_2}$$

Note: After a short time, current will no longer flow through a C, and any segment of a circuit with a C will have no current.

Kirchoff’s Rules

Junction Rule: $I_{in} = I_{out}$

- Total current flowing into junction equals total current flowing out of junction

Loop Rule: $\Sigma \Delta V = 0$

- The sum of the changes in voltage around any closed loop always equals zero

$\Delta V = -IR$ (if you **pass through** resistor in the **same direction as current**)

$\Delta V = IR$ (if you **pass through** resistor in the **opp. direction as current**)

$\Delta V = +\varepsilon_{battery}$ (if you **pass through** the battery from – **terminal to** + **terminal**)

$\Delta V = -\varepsilon_{battery}$ (if you **pass through** the battery from + **terminal to - terminal**)

Terminal Voltage

$V_{ab} = \varepsilon - Ir$ (V_{ab} is the terminal voltage, ε is the emf of battery, **r** is internal resistance)

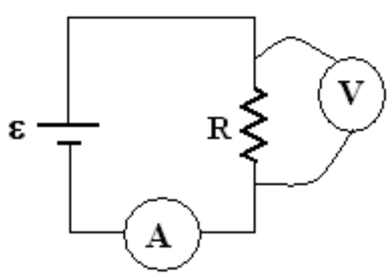
-Every battery has an internal resistance **r** which will lower the terminal voltage when current flows

- A 9V battery will not necessarily have a measured terminal voltage of 9V, unless no current flows
- The ε of a 9V battery is 9V even when no current flows, but the measured terminal voltage will be less
- **Slope** of Vab vs. I graph is negative the **internal resistance**. The **y intercept** is the **emf** ε.

Electrical Meters

Voltmeter

- Measures voltage change across circuit element (resistor, battery, etc.)
- Ideally has **infinite resistance** so it does not draw any current away from circuit
- Needs to be hooked up in **parallel** with circuit element



Ammeter

- Measures current through a circuit element (resistor, battery, etc.)
- Ideally has **no resistance** so it does not change the current
- Needs to be hooked up in **series** with circuit element

Magnetism

Magnetic forces

$F_B = qvB\sin\theta_{vB}$ (**q** is charge, v is speed, **B** is magnetic field, θ is angle between v and B)

- The direction of force on + charge is given by the **Right hand rule** (**V**ery-**B**ad-**F**inger)
- If the charge is negative the force is in the opposite direction
- Magnetic forces <u>**never do Work**</u> (since F_B is always **perpendicular** to motion W=Fdcos**90**=0)
- Magnetic forces often make charges (**q**) of mass **m** travel in circles of radius **r** given by,

$r = mv/qB$

Note: If you want a charged particle to travel in a **straight line ("velocity selector")**, create an electric field E so that the forces cancel, i.e. speed is ratio of E to B

$v=E/B$ (since $F_B = F_E$ or $qvB = qE$)

Note: The forces have to be of equal size, <u>not the fields!</u> (i.e. $\mathbf{F_B = F_E}$, but **E** does not equal **B**)

<u>Magnetic force on wire</u>

$F_B = ILB$ (I is current that <u>feels force</u>, L is length of wire, B is mag. field <u>from other sources</u>)

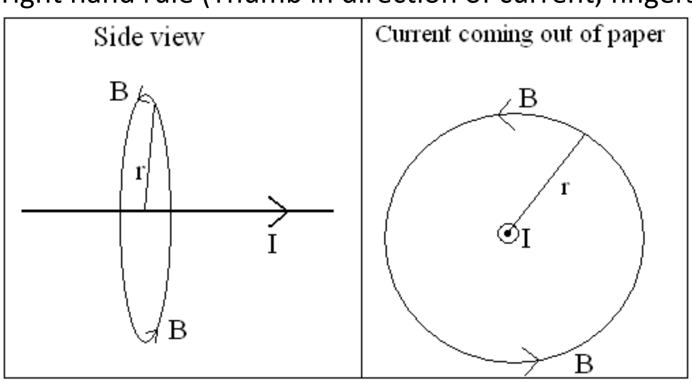
-to find direction of F_B use the same right hand rule (except **v** is now direction of **I**)

Magnetic fields

The magnetic field a distance **r** from a long straight current (**I**) carrying wire is given by,

$B = \mu_o I/2\pi r$ [Tesla] $\mu_o = 4\pi \times 10^{-7}$ T m/A

- The magnetic field from a long straight wire is directed along a circle centered at wire with direction given by right hand rule (Thumb in direction of current, fingers curl in direction of B)



- Note: Wires with I in same direction will attract, Wires with I in opp. Direction will repel