Write the quadratic function with the given complex root(s) and a point on the parabola.

1. roots : $\pm 4i$ passes through $(2, -5)$

$r_1 = 4i \quad r_2 = -4i$

Root Form:

$y = a(x-4i)(x+4i)$

$= a(x^2 + 4ix - 4ix - 16i^2)$

$= a(x^2 - 16i^2)$ but $i^2 = -1$

$= a(x^2 - 16(-1))$

Eq.1 $y = a(x^2+16)$

To solve for "a":

@ Pt $(\overset{x}{2}, \overset{y}{-5})$

$-5 = a(2^2+16)$

$-5 = a(4+16)$

$\frac{-5}{20} = \frac{a(20)}{20}$

$-\frac{1}{4} = a$

Substitute $a = -\frac{1}{4}$ in Eq. 1

$y = -\frac{1}{4}(x^2+16)$

$\boxed{y = -\frac{1}{4}x^2 - 4}$

2. one root : $5-3i$ passes through $(8, -30)$

$r_1 = 5-3i$

$r_2 = 5+3i$ (r_2 is conjugate of r_1)

Root Form

$y = a(x-r_1)(x-r_2)$

$= a(x-5+3i)(x-5-3i)$

$= a[(x-5)^2 - 9i^2]$

$= a[(x^2-10x+25) - 9i^2]$ $\quad i^2 = -1$

$= a[x^2 - 10x + 25 - 9(-1)]$

$= a(x^2 - 10x + 25 + 9)$

Eq.1 $y = a(x^2 - 10x + 34)$

To solve for "a", use Pt $(8, -30)$:

$-30 = a(8^2 - 10(8) + 34)$

$= a(64 - 80 + 34)$

$\frac{-30}{18} = \frac{a(18)}{18}$

$-\frac{5}{3} = a$

Substitute $a = -\frac{5}{3}$ to Eq. 1

$y = -\frac{5}{3}(x^2 - 10x + 34)$

$\boxed{y = -\frac{5}{3}x^2 + \frac{50}{3}x - \frac{170}{3}}$

3. roots : $1 \pm i\sqrt{2}$ passes through $(2, -3)$

$r_1 = 1 + i\sqrt{2}$

$r_2 = 1 - i\sqrt{2}$

Root Form

$y = a(x - 1 - i\sqrt{2})(x - 1 + i\sqrt{2})$

$= a[(x-1)^2 - i^2(2)]$

$= a[(x^2 - 2x + 1) - (-1)(2)]$

$= a[x^2 - 2x + 1 + 2]$

Eq. 1 $y = a(x^2 - 2x + 3)$

→ To solve for a, use Pt $(2, -3)$ (x, y)

$-3 = a(2^2 - 2(2) + 3)$

$\frac{-3}{3} = \frac{a(3)}{3}$

$-1 = a$ ✓✓

Substitute $a = -1$ in Eq. 1

$y = -1(x^2 - 2x + 3)$

$\boxed{y = -x^2 + 2x + 3}$

4. one root : $-i$ passes through $(-3, 20)$

$r_1 = -i$

$r_2 = i$ (conjugate)

Root Form

$y = a(x + i)(x - i)$

$y = a(x^2 - i^2)$ note: $i^2 = -1$

Eq. 1 $y = a(x^2 + 1)$

To solve for "a" use Pt $(-3, 20)$ (x, y) ✗ ✓

$20 = a((-3)^2 + 1)$

$20 = a(9 + 1)$

$\frac{20}{10} = \frac{a(10)}{10}$

$2 = a$

→ Substitute $a = 2$ in Eq. 1

$y = 2(x^2 + 1)$

$\boxed{y = 2x^2 + 2}$