AP* Statistics Test A – Inference for Proportions – Part V – Key

1. B 2. D 3. E 4. E 5. D 6. D 7. B 8. B 9. E 10. E

11. President's approval rating: The smaller poll would have more variability and would thus be more likely to vary from the actual approval rating of 54%. We would expect the larger poll to be more consistent with the 54% rating. So, it is more likely that the smaller poll would report that the President's approval rating is below 50%.

12. Cereal: Two methods can be used to solve this problem:

Method 1:

Let B = weight of one box of cereal and T = weight of 12 boxes of cereal.

We are told that the contents of the boxes are approximately Normal, and we can assume that the content amounts are independent from box to box.

$E(\mathrm{T}) = E(\mathrm{B}_1 + \mathrm{B}_2 + \cdots + \mathrm{B}_{12}) = E(\mathrm{B}_1) + E(\mathrm{B}_2) + \cdots + E(\mathrm{B}_{12}) = 156$ ounces

Since the content amounts are independent,

$Var(\mathrm{T}) = Var(\mathrm{B}_1 + \mathrm{B}_2 + \cdots + \mathrm{B}_{12}) = Var(\mathrm{B}_1) + Var(\mathrm{B}_2) + \cdots + Var(\mathrm{B}_{12}) = 3$

$SD(\mathrm{T}) = \sqrt{Var(\mathrm{T})} = \sqrt{3} = 1.73$ ounces

We model T with $N(156, 1.73)$.

$z = \dfrac{160-156}{1.73} = 2.31$ and $P(\mathrm{T} > 160) = P(z > 2.31) = 0.0104$

There is a 1.04% chance that a case of 12 cereal boxes will weigh more than 160 ounces.

Method 2:

Using the Central Limit Theorem approach, let $\bar{y}$ = average content of boxes in the case. Since the contents are Normally distributed, $\bar{y}$ is modeled by $N\left(13, \dfrac{0.5}{\sqrt{12}}\right)$.

$$P\left(\bar{y} > \frac{160}{12}\right) = P(\bar{y} > 13.33) = P\left(z > \frac{13.33-13}{0.5/\sqrt{12}}\right) = P(z > 2.31) = 0.0104.$$

There is a 1.04% chance that a case of 12 cereal boxes will weigh more than 160 ounces.

13. Exercise: A random sample of 150 men found that 88 of the men exercise regularly, while a random sample of 200 women found that 130 of the women exercise regularly.

a. Conditions:

* Randomization Condition: We are told that we have random samples.
* 10% Condition: We have less than 10% of all men and less than 10% of all women.
* Independent samples condition: The two groups are clearly independent of each other.
* Success/Failure Condition: Of the men, 88 exercise regularly and 62 do not; of the women, 130 exercise regularly and 70 do not. The observed number of both successes and failures in both groups is at least 10.

With the conditions satisfied, the sampling distribution of the difference in proportions is approximately Normal with a mean of $p_M - p_W$, the true difference between the population proportions. We can find a two-proportion z-interval.

We know: $n_M = 150$, $\hat{p}_M = \frac{88}{150} = 0.587$, $n_W = 200$, $\hat{p}_W = \frac{130}{200} = 0.650$.

We estimate $SD(\hat{p}_M - \hat{p}_W)$ as

$$SE(\hat{p}_M - \hat{p}_W) = \sqrt{\frac{\hat{p}_M \hat{q}_M}{n_M} + \frac{\hat{p}_W \hat{q}_W}{n_W}} = \sqrt{\frac{(0.587)(0.413)}{150} + \frac{(0.65)(0.35)}{200}} = 0.0525$$

$$ME = z^* \times SE(\hat{p}_M - \hat{p}_W) = 1.96(0.0525) = 0.1029$$

The observed difference in sample proportions = $\hat{p}_M - \hat{p}_W = 0.587 - 0.650 = -0.063$, so the 95% confidence interval is -0.063 ± 0.1029, or -16.6% to 4.0%.
We are 95% confident that the proportion of women who exercise regularly is between 4.0% lower and 16.6% higher than the proportion of men who exercise regularly.

b. Since zero is contained in my confidence interval, I cannot say that a higher proportion of women than men exercise regularly. My confidence interval does not support my friend's claim.

14. Internet access:

Since $ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$, we have or $z^* = \frac{0.03}{\sqrt{\frac{(0.28)(0.72)}{1028}}} \approx 2.14$.

Our confidence level is approximately $P(-2.14 < z < 2.14) = 0.9676$, or 97%.

15. Sleep

Hypothesis: $H_0: p = 0.50 \quad H_A: p > 0.50$

Plan: Okay to use the Normal model because the trials are independent (random sample of U.S. adults), these 1003 U.S. adults are less than 10% of all U.S. adults, and $np_0 = (1003)(0.50) = 501.5 \geq 10$ and $nq_0 = (1003)(0.50) = 501.5 \geq 10$.

We will do a one-proportion z-test.

Mechanics: $SD(p_0) = \sqrt{\frac{p_0 q_0}{n}} = \sqrt{\frac{(0.50)(0.50)}{1003}} = 0.0158$; sample proportion: $\hat{p} = 0.55$

$$P(\hat{p} > 0.55) = P(z > \frac{0.55 - 0.50}{0.0158}) = P(z > 3.16) = 0.0008$$

With a *P*-value of 0.0008, I reject the null hypothesis. There is strong evidence that the proportion of U.S. adults who feel they get enough sleep is more than 50%.