Remember: $base^{exponent} = answer$ | $\ln = \log_e$

$\log_{base} answer = exponent$

Algebra II

Name _____

Review - Exponential & Logarithmic Functions

Date _____ per _____

*Write each equation in log form.*

1. $5^3 = 125$ $\quad \log_5 125 = 3$

2. $27^{\frac{4}{3}} = 81$ $\quad \log_{27} 81 = \frac{4}{3}$

*Write each equation in exponential form*

3. $\log 0.00001 = -5$ $\quad 10^{-5} = 0.00001$ (base = 10)

4. $\log_{\frac{3}{2}} \frac{\sqrt{6}}{3} = -\frac{1}{2}$ $\quad \left(\frac{3}{2}\right)^{-\frac{1}{2}} = \frac{\sqrt{6}}{3}$

*Write the inverse of each of the following functions.*

5. $f(x) = \log_5 x + 2$

$y = \log_5 x + 2$

$x = \log_5 y + 2$

$5^x = y + 2$

$5^x - 2 = y$

6. $f(x) = 3^{x-4} - 6$ (move −6 to left side)

$y = 3^{x-4} - 6$

$x = 3^{y-4} - 6$

$x + 6 = 3^{y-4}$

$\log_3 (x+6) = y - 4$

$\log_3 (x+6) + 4 = y$

7. $f(x) = -3\ln x + 4$

$y = -3\ln x + 4$

$x = -3\ln y + 4$

$x - 4 = -3\ln y$

$\frac{x-4}{-3} = \ln y$

$e^{\frac{x-4}{-3}} = y$

8. $f(x) = \left(\frac{1}{2}\right)^{-x+1} - 6$

$y = \left(\frac{1}{2}\right)^{-x+1} - 6$

$x = \left(\frac{1}{2}\right)^{-y+1} - 6$

$x + 6 = \left(\frac{1}{2}\right)^{-y+1}$

$\log_{\frac{1}{2}} (x+6) = -y + 1$

$-\log_{\frac{1}{2}} (x+6) + 1 = y$

*Evaluate the following.*

9. $\log_4 64 = 3$

$4^x = 64$

$4^x = (4^3)$

$x = 3$

10. $\log \frac{1}{10} = -1$ (base = 10)

$10^x = \frac{1}{10}$

$10^x = 10^{-1}$

$x = -1$

11. $\log_4 \frac{1}{16} = -2$

Note: $\frac{1}{16} = \frac{1}{4^2} = 4^{-2}$

$4^x = \frac{1}{16}$

$4^x = (4^{-2})$

$x = -2$

12. $\log_{1/2} 4\sqrt{2} =$

$\left(\frac{1}{2}\right)^x = (2^2)\left(2^{\frac{1}{2}}\right)$

$(2^{-1})^x = 2^{2\frac{1}{2}}$

$x = -2\frac{1}{2}$

13. $\log_{1/25} \sqrt{125} = -\frac{3}{4}$

$\left(\frac{1}{25}\right)^x = \sqrt{125}$

$(5^{-2})^x = \sqrt{(5^3)}$

$5^{-2x} = 5^{\frac{3}{2}}$

$-2x\left(\frac{1}{-2}\right) = \frac{3}{2}\left(\frac{1}{-2}\right)$

$x = -\frac{3}{4}$

14. $\log_{27} \left(\frac{\sqrt{3}}{3}\right) =$

$27^x = \frac{\sqrt{3}}{3}$

$(3^3)^x = 3^{\frac{1}{2}-1}$

$\left(\frac{1}{3}\right) 3x = -\frac{1}{2}\left(\frac{1}{3}\right)$

$x = -\frac{1}{6}$

Note: $\frac{\sqrt{3}}{3} = \frac{3^{\frac{1}{2}}}{3^1} = 3^{\frac{1}{2}-1}$

*Solve for x.*

15. $\log_3(3x-5)=2$

$3^2 = 3x-5$

$9 = 3x-5$

$+5 \quad +5$

$14 = 3x$

$\frac{14}{3} = \frac{3x}{3}$

$x = \frac{14}{3}$

16. $\log\frac{\sqrt{10}}{10} = x-3$

$10^{x-3} = \frac{\sqrt{10}}{10}$

$10^{x-3} = 10^{\frac{1}{2}} \cdot 10^{-1}$

$10^{x-3} = 10^{-\frac{1}{2}}$

$x-3 = -\frac{1}{2}$

$+3 \quad +3$

$x = 2.5$

17. $\log_{17} 17^{(2x+3)} = 4$

$17^4 = 17^{2x+3}$

$4 = 2x+3$

$-3 \quad -3$

$1 = 2x$

$\frac{1}{2} = \frac{2x}{2}$

$x = \frac{1}{2}$

18. $\log_x 27\sqrt[3]{9} = \frac{1}{6}$

$x^{\frac{1}{6}} = 27\cdot\sqrt[3]{9}$

$x^{\frac{1}{6}} = 3^3 \cdot (3^2)^{\frac{1}{3}}$

$x^{\frac{1}{6}} = 3^3 \cdot 3^{\frac{2}{3}}$

$x^{\frac{1}{6}} = 3^{\frac{11}{3}}$

$(x^{\frac{1}{6}})^6 = (3^{\frac{11}{3}})^6$

$x = 3^{22}$

$x = 31381059609$

19. $7^{6x} = 7^{2x-20}$

$6x = 2x-20$

$-2x \quad -2x$

$4x = -20$

$\frac{4x}{4} = \frac{-20}{4}$

$x = -5$

20. $4^{3x-1} \cdot 2 = \left(\frac{1}{8}\right)^x$

$(2^2)^{3x-1} \cdot 2^1 = (8^{-1})^x$

$2^{6x-2} \cdot 2^1 = 8^{-x}$

$2^{6x-1} = (2^3)^{-x}$

$2^{6x-1} = 2^{-3x}$

$6x-1 = -3x$

$+3x+1 \quad +3x+1$

$9x = 1$

$\frac{9x}{9} = \frac{1}{9}$

$x = \frac{1}{9}$

21. $4\log_2 x + \log_2 5 = \log_2 80$

22. $\log_8(x-3) + \log_8(x+4) = 1$

23. $9^{2x-1} = 27^{x+4}$

$(3^2)^{2x-1} = (3^3)^{x+4}$

$3^{4x-2} = 3^{3x+12}$

$4x-2 = 3x+12$

$-3x+2 \quad -3x+2$

$x = 14$

$x = 14$

24. $\log_9 \frac{(3x+14)}{5} = \log_9 2x$

$5 \cdot \frac{3x+14}{5} = 2x \cdot 5$

$3x+14 = 10x$

$-3x \quad -3x$

$14 = 7x$

$\frac{14}{7} = \frac{7x}{7}$

$x = 2$

25. $\left(\frac{1}{16}\right)^{x+1} = \left(\frac{1}{8}\right)^{2x-1}$

$(16^{-1})^{x+1} = (8^{-1})^{2x-1}$

$(2^4)^{-x-1} = (2^3)^{-2x+2}$

$2^{-4x-4} = 2^{-6x+3}$

$-4x-4 = -6x+3$

$+6x+4 \quad +6x+4$

$2x = 7$

$\frac{2x}{2} = \frac{7}{2}$

$x = \frac{7}{2}$

26. $5^{2x+3} = \left(\sqrt{5}\right)^{x+4}$

$5^{2x+3} = (5^{\frac{1}{2}})^{x+4}$

$5^{2x+3} = 5^{\frac{1}{2}x+2}$

$2x+3 = \frac{1}{2}x+2$

$-\frac{1}{2}x \cdot 2 \quad -\frac{1}{2}x \cdot 2$

$1.5x = -1$

$\frac{1.5x}{1.5} = \frac{-1}{1.5}$

$x = -\frac{2}{3}$

27. $\log_7 x^{\frac{3}{2}} = \log_7 8$

$(x^{\frac{3}{2}})^{\frac{2}{3}} = (8)^{\frac{2}{3}}$

$x = 8^{\frac{2}{3}} \Rightarrow x = 4$

28. $2\cdot 10^{2x-3}-4=196$

$2(10)^{2x-3}-4=196$

$+4 \quad +4$

$\frac{2(10)^{2x-3}}{2}=\frac{200}{2}$

$10^{2x-3}=100$

$10^{2x-3}=(10^{10})$

$2x-3=10$

$2x=13$

$x=\frac{13}{2}$

29. $16^{3x+1}=8$

$(2^4)^{3x+1}=(2^3)$

$12x+4=3$

$12x=-1$

$x=\frac{-1}{12}$

30. $4^{2x}=32$

$(2^2)^{2x}=(2^5)$

$4x=5$

$x=\frac{5}{4}$

31. $e^{x+1}=4$

$\ln[e^{x+1}]=\ln[4]$

$x+1=\ln 4$

$-1 \quad -1$

$x=\ln 4-1$

Graph the following and provide the other information requested.

**32.** $f(x)=\log_2(x+2)-4$

a) Domain $(-2,\infty)$

b) Range $\mathbb{R}$

c) x intercept $14$

d) y intercept $-3$

e) asymptote $x=-2$

f) increasing or decreasing

Transformations:

H. shift left by 2

V. shift down by 4

x-int ($y=0$)

$0=\log_2(x+2)-4$

$4=\log_2(x+2)$

$2^4=x+2$

$16=x+2$

$14=x$ ✓

y-int

$y=\log_2 2-4$

$=1-4$

$y=-3$ ✓

PF

| x | y |
|---|---|
| $\frac{1}{2}$ | -1 |
| 1 | 0 |
| 2 | 1 |

GF

| x | y |
|---|---|
| $-1\frac{1}{2}$ | -5 |
| -1 | -4 |
| 0 | -3 |

**33.** $y=2\left(\frac{1}{3}\right)^{x-3}-9$

a) Domain $\mathbb{R}$

b) Range $(-9,\infty)$

c) x intercept $\log_{\frac{1}{3}}\left(\frac{9}{2}\right)+3$

d) y intercept

e) asymptote $x=-9$

Transformations:

- vert. stretch by 2
- right by 3
- down by 9

x-int ($y=0$)

$0=2\left(\frac{1}{3}\right)^{x-3}-9$

$9=2\left(\frac{1}{3}\right)^{x-3}$

$\frac{9}{2}=\left(\frac{1}{3}\right)^{x-3}$

$x-3=\log_{\frac{1}{3}}\left(\frac{9}{2}\right)$

$x=\log_{\frac{1}{3}}\left(\frac{9}{2}\right)+3$

y-int ($x=0$)

$y=2\left(\frac{1}{3}\right)^{0-3}-9$

GF

| x | y |
|---|---|
| 1 | 9 |
| 2 | -3 |
| 3 | -7 |

f) increasing or decreasing

**34.** $f(x)=\ln(x+4)$

a) Domain $(-4, \infty)$

b) Range $\mathbb{R}$

c) x intercept $-3$

d) y intercept $1.4$

e) asymptote $x=-4$

f) increasing or decreasing

Transformations:

H. shift left by 4

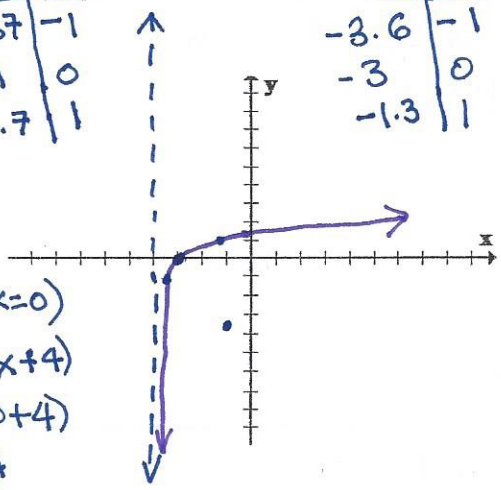
$x = e^y$

PF

| x | y |
|---|---|
| 0.37 | -1 |
| 1 | 0 |
| 2.7 | 1 |

GF

| x | y |
|---|---|
| -3.6 | -1 |
| -3 | 0 |
| -1.3 | 1 |




x-int ($y=0$)

$0 = \ln(x+4)$

$e^0 = x+4$

$1 = x+4$

$-3 = x$

y-int ($x=0$)

$y = \ln(x+4)$

$= \ln(0+4)$

$= \ln 4$

$y = 1.4$

**35.** $f(x)=-3\cdot 2^{x-1}+6$ ✓

a) Domain $\mathbb{R}$

b) Range $(-\infty, 6)$

c) x intercept $2$

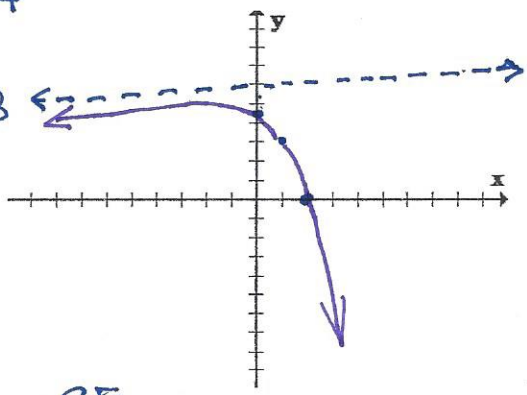
d) y intercept $4.5$

e) asymptote $y = 6$

f) increasing or decreasing

Transformations:

- Reflection over x
- Vertical stretch by 3
- H. shift right by 1
- Up by 6




$y = 2^x$

PF

| x | y |
|---|---|
| -1 | $\frac{1}{2}$ |
| 0 | 1 |
| 1 | 2 |

$-\frac{1}{2} \to -1\frac{1}{2}$

$-1 \to -3$

$-2 \to -6$

reflection x-axis

vertical stretch by 3

GF

| x | y |
|---|---|
| 0 | 4.5 |
| 1 | 3 |
| 2 | 0 |

x-int. ($y=0$)

$y = -3(2)^{x-1}+6$

$0 = -3(2)^{x-1}+6$

$-6 \quad\quad -6$

$\frac{-6}{-3} = \frac{-3(2)^{x-1}}{-3}$

$2^1 = 2^{x-1}$

$1 = x-1$

$2 = x$

y-int. ($x=0$)

$y = -3(2)^{x-1}+6$

$= -3(2)^{0-1}+6$

$= -3(2^{-1})+6$

$y = -\frac{3}{2}+6$

$y = 4.5$ or $4\frac{1}{2}$