

Review 11.1, 11.2 & 11.6 - Solving Rational Equations, Applications & Graphing

State the restrictions and solve each rational equation.

Restrictions $x \neq -2$ 1. $\frac{3}{x+2} = \frac{x-3}{2x+4}$

Solution(s) $x = 9$

$$\frac{3}{x+2} \cdot \frac{2}{2} - \frac{x-3}{2(x+2)} = 0$$

$$\frac{6}{2(x+2)} - \frac{x-3}{2(x+2)} = 0$$

$$6 - x + 3 = 0$$

$$x = 9$$

Restrictions $x \neq -6, 0$ 2. $\frac{1}{x+6} + \frac{x+1}{x} = \frac{13}{x+6}$

Solution(s) $x = 3, 2$

$$\frac{1}{x+6} \cdot \frac{x}{x} + \frac{x+1}{x} \cdot \frac{x+6}{x+6} - \frac{13}{x+6} \cdot \frac{x}{x} = 0$$

$$\underline{x} + x^2 + 7x + 6 - 13x = 0$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x = 3 \quad | \quad x = 2$$

Restrictions $x \neq -4$ 3. $\frac{2x+14}{x+4} - 2 = \frac{2x+20}{2x+8}$

Solution(s) $x = -20$

$$\frac{2x+14}{x+4} \cdot \frac{2}{2} - 2 \cdot \frac{2(x+4)}{2(x+4)} - \frac{2x+20}{2(x+4)} = 0$$

$$\underline{8x} + \underline{28} - \underline{4x} - \underline{16} - \underline{2x} - \underline{20} = 0$$

$$2x + 2 = 0$$

$$2x = -2$$

$$x = -1$$

Restrictions $x \neq 1, -1$

Solution(s) $x = 5, -2$

4. $\frac{3x}{x+1} = \frac{12}{x^2-1} + 2$

$$\begin{aligned} \frac{3x}{x+1} \cdot \frac{x-1}{x-1} - \frac{12}{(x+1)(x-1)} - 2 \frac{(x+1)(x-1)}{(x+1)(x-1)} &= 0 \\ 3x^2 - 3x - 12 - 2x^2 + 2 &= 0 \\ x^2 - 3x - 10 &= 0 \\ (x-5)(x+2) &= 0 \\ x=5 \mid x=-2 \end{aligned}$$

Restrictions $x \neq 4, 3$

Solution(s) $x = -2$

5. $\frac{3x}{x-4} - \frac{2x}{x-3} = \frac{6}{x^2-7x+12}$

$x^2 - 7x + 12 = (x-3)(x-4)$

$$\begin{aligned} \frac{3x}{x-4} \cdot \frac{x-3}{x-3} - \frac{2x}{x-3} \cdot \frac{x-4}{x-4} - \frac{6}{(x-3)(x-4)} &= 0 \\ 3x^2 - 9x - 2x^2 + 8x - 6 &= 0 \\ x^2 - x - 6 &= 0 \\ (x-3)(x+2) &= 0 \\ x=3 \mid x=-2 \end{aligned}$$

$\hookrightarrow$ can't be a solution

Restrictions $x \neq -2$

Solution(s) no solution

6. $\frac{x+3}{x+2} = 2 - \frac{x+1}{x+2}$

$$\begin{aligned} \frac{x+3}{x+2} - 2 \frac{x+2}{x+2} + \frac{x+1}{x+2} &= 0 \\ \underline{x+3} - \underline{2x-4} + \underline{x+1} &= 0 \\ 0 &= 0 \end{aligned}$$

Define the variable, write a rational equation and solve. Label your solution.

7. Fran can mow a lawn in 45 minutes. Gail can mow a lawn in one hour. Working together, how long will it take to complete the job?

	Work Rate	Time	Amt. of work
Fran	$\frac{1}{45}$	x	$\frac{x}{45}$
Gail	$\frac{1}{60}$	x	$\frac{x}{60}$

$$\text{Work}_{\text{Fran}} + \text{Work}_{\text{Gail}} = 1$$

$$\frac{x}{45} + \frac{x}{60} = 1 \quad \text{LCD} = 180$$

$$\frac{x}{45} \cdot \frac{4}{4} + \frac{x}{60} \cdot \frac{3}{3} = 1 \cdot \frac{180}{180}$$

$$4x + 3x = 180$$

$$7x = 180$$

$$x = \frac{180}{7} \text{ or } 25.7 \text{ min.}$$

8. Bob paints the house in 10 days. If Sam helps the house can be painted in 6 days. How many days would it take Sam to paint the house alone?

	Work Rate	Time	Amt. of work
Bob	$\frac{1}{10}$	6	$\frac{6}{10}$
Sam	$\frac{1}{x}$	6	$\frac{6}{x}$

$$\frac{6}{10} + \frac{6}{x} = 1$$

$$\frac{6}{10} \cdot \frac{x}{x} + \frac{6}{x} \cdot \frac{10}{10} = 1 \cdot \frac{10x}{10x}$$

$$\frac{6x}{10} + \frac{60}{x} = \frac{10x}{10x}$$

$$\frac{-4x}{10} + \frac{60}{x} = \frac{0}{10x}$$

$$\frac{-4x}{10} = \frac{-60}{x}$$

$$x = 15 \text{ days}$$

9. There are three water pumps (pump A, pump B and pump C) used to fill a pool. Pump A can fill the pool in 6 hours. Pump B takes an entire day (24 hours) to fill the pool. If it takes Pump A, B & C three hours to fill the pool together, how long will it take Pump C to fill the pool by itself?

	Work Rate	Time	Amt. of work
A	$\frac{1}{6}$	3	$\frac{3}{6}$
B	$\frac{1}{24}$	3	$\frac{3}{24}$
C	$\frac{1}{x}$	3	$\frac{3}{x}$

$$\frac{3}{6} + \frac{3}{24} + \frac{3}{x} = 1$$

$$\frac{3}{6} \cdot \frac{4x}{4x} + \frac{3}{24} \cdot \frac{x}{x} + \frac{3}{x} \cdot \frac{24}{24} = 1 \cdot \frac{24x}{24x}$$

$$12x + 3x + 72 = 24x$$

$$12x + 3x - 24x = -72$$

$$-9x = -72$$

$$x = 8 \text{ hours}$$

10. Ken can fill the potato bin at HEB in 20 minutes. Kristen can fill the same bin in 25 minutes. If Ken and Kristen work together, how fast can they fill the potato bin?

Ken	$\frac{1}{20}$	$\times$	$\frac{x}{20}$
Kristin	$\frac{1}{25}$	$\times$	$\frac{x}{25}$

$$\frac{x}{20} + \frac{x}{25} = 1$$

$$\frac{x}{20} \cdot \frac{5}{5} + \frac{x}{25} \cdot \frac{4}{4} = 1 \cdot \frac{100}{100}$$

$$5x + 4x = 100$$

$$9x = 100$$

$$x = 11.11 \text{ min.}$$

11. Dylan can row 30 km downstream in the same amount of time he can row 18 km upstream. The rate of the current is 2 km/h. Find Dylan's rate.

	Distance	Rate	Time
Downstream	30	$x+2$	$\frac{30}{x+2}$
Upstream	18	$x-2$	$\frac{18}{x-2}$

$$\text{Time downstream} = \text{Time upstream}$$

$$\frac{30}{x+2} = \frac{18}{x-2}$$

$$\frac{30}{x+2} \cdot \frac{x-2}{x-2} - \frac{18}{x-2} \cdot \frac{x+2}{x+2} = 0$$

$$30x - 60 - 18x - 36 = 0$$

$$12x - 96 = 0$$

$$12x = 96$$

$$x = 8 \frac{\text{km}}{\text{h}}$$

12. A boat, which moves at 36 mi/h in still water, travels 28 mi downstream in the same amount of time that it takes to travel 20 mi upstream. Find the speed of the current.

	Distance	Rate	Time
Downstream	28	$36+x$	$\frac{28}{36+x}$
Upstream	20	$36-x$	$\frac{20}{36-x}$

$$\frac{28}{36+x} = \frac{20}{36-x}$$

$$\frac{28}{36+x} \cdot \frac{36-x}{36-x} - \frac{20}{36-x} \cdot \frac{36+x}{36+x} = 0$$

$$1008 - 28x - 720 - 20x = 0$$

$$288 - 48x = 0$$

$$\begin{array}{r} -48x \\ \hline -48 \end{array} \quad \begin{array}{r} = -288 \\ \hline -48 \end{array}$$

$$x = 6 \text{ mph}$$

13. A small jet has an airspeed (the rate in still air) of 300 mi/h. During one day's flights, the pilot noted that the plane could fly 85 mi with a tailwind in the same time it took to fly 65 mi against that same wind. What was the rate of the wind?

	Distance	Rate	Time
Tailwind	85	$300 + x$	$\frac{85}{300 + x}$
Against	65	$300 - x$	$\frac{65}{300 - x}$

$$\frac{85}{300 + x} = \frac{65}{300 - x}$$

$$\frac{85}{300 + x} \cdot \frac{300 - x}{300 - x} - \frac{65}{300 - x} \cdot \frac{300 + x}{300 + x} = 0$$

$$25,500 - 85x - 19,500 - 65x = 0$$

$$-150x + 6000 = 0$$

$$-150x = -6000$$

$$x = 40 \text{ mph}$$

14. You have 100 mL of a solution that is 30% acid. How many mL of acid do you need to add so that your new mixture is 60% acid?

	% (dec.)	Volume	Amount Acid
30% soln.	0.30	100	30
Acid (Pure)	1.00	x	x
Mixture	0.60	$100 + x$	$30 + x$

75 mL of Acid (pure)

$$0.60(100 + x) = 30 + x$$

$$60 + 0.60x = 30 + x$$

$$\begin{array}{r} 60 + 0.60x \\ - 0.60x \\ \hline 60 \end{array} = \begin{array}{r} 30 + x \\ - 0.60x \\ \hline 30 + 0.40x \end{array}$$

$$\begin{array}{r} 60 \\ - 30 \\ \hline 30 \end{array} = \begin{array}{r} 30 + 0.40x \\ - 30 \\ \hline 0.40x \end{array}$$

$$75 \text{ mL} = x$$

15. A pharmacist wishes to make 1.8 L of a 10% solution of boric acid by mixing 7.5% and 12% solutions. How much of each type of solution should be used?

7.5% soln.	0.075	x	$0.075x$
12% soln.	0.12	$1.8 - x$	$0.216 - 0.12x$
Mixture	0.10	1.8	0.18

0.8 L of 7.5% soln. and
1.0 L of 12% soln.

$$0.075x + (0.216 - 0.12x) = 0.18$$

$$0.075x + 0.216 - 0.12x = 0.18$$

$$0.216 - 0.045x = 0.18$$

$$\begin{array}{r} 0.216 - 0.045x \\ - 0.216 \\ \hline -0.045x = -0.036 \end{array}$$

$$\begin{array}{r} -0.045x = -0.036 \\ -0.045 \\ \hline x = 0.8 \text{ L} \end{array}$$

$$1.8 - x = 1.0 \text{ L}$$

16. Two hundred liters of a punch that contains 35% fruit juice is mixed with 300 liters (L) of another punch. The resulting fruit punch is 20% fruit juice. Find the percent of fruit juice in the 300 liters of punch?

	%	Vol.	Amount Fruit J
Punch A	0.35	200	70
Punch B	x	300	300x
Mixture	0.20	500	100

$$\begin{array}{r}
 70 + 300x = 100 \\
 -70 \quad \quad -70 \\
 \hline
 300x = 30 \\
 \frac{300x}{300} = \frac{30}{300}
 \end{array}$$

$$x = 0.10 \text{ or } 10\%$$

Punch B has 10% fruit juice

17. Find y when x = 76, if y varies inversely as x and y = 100, when x = 35.

$$\begin{aligned}
 y &= \frac{k}{x} \\
 @ (35, 100) \\
 100 &= \frac{k}{35} \\
 3500 &= k \\
 y &= \frac{3500}{x} \\
 y &= \frac{3500}{76} \\
 y &= 46.05
 \end{aligned}$$

18. If y varies inversely as x and y = 3 when x = 6, find x when y = 18.

$$\begin{aligned}
 y &= \frac{k}{x} \\
 @ (6, 3) \\
 3 &= \frac{k}{6} \\
 18 &= k \\
 y &= \frac{18}{x} \\
 \text{when } y &= 18 \\
 18 &= \frac{18}{x} \\
 x &= \frac{18}{18} \\
 x &= 1
 \end{aligned}$$

19. The time to drive a certain distance varies inversely according to the rate of speed. Mary drives 47 mph for 4 hours. How long would it take her to make the same trip at 55 mph?

$$\begin{aligned}
 \text{time} &= \frac{k}{\text{rate}} \\
 4 &= \frac{k}{47} \\
 188 &= k \\
 \text{time} &= \frac{188}{\text{rate}} \\
 &= \frac{188}{55} \\
 \text{time} &= 3.42 \text{ hr}
 \end{aligned}$$

20. The intensity of light, measured in *lux*, is inversely proportional to the square of the distance between the light source and the object illuminated. A light meter 7.5 m from a light source registers 24 *lux*. What intensity would it register 15 m from the light source?

$$I = \frac{K}{d^2} \quad K = 1350 \quad I = \frac{1350}{15^2}$$

$$24 = \frac{K}{7.5^2} \quad I = \frac{1350}{d^2} \quad \boxed{I = 6 \text{ lux}}$$

$$24 = \frac{K}{56.25}$$

21. The cost per person to rent a chartered bus varies inversely to the number of people who ride the bus. If 40 people pay \$9.50 each to ride the chartered bus, what is the cost per person if only 25 people go?

$$C = \frac{K}{N} \quad C = \frac{380}{N}$$

$$9.50 = \frac{K}{40} \quad C = \frac{380}{25}$$

$$K = 380 \quad \boxed{C = \$15.20}$$

Graph each of the following rational functions.

22) $f(x) = \frac{1}{x+2} + 4$

x-Int. $-\frac{9}{4}$
(y=0)

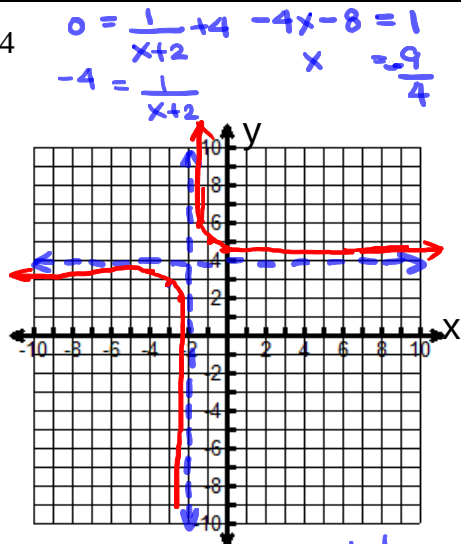
y-int. $4\frac{1}{2}$
(x=0)

V.A. $x = -2$

H.A. $y = 4$

Transformations:

Left by 2
Up by 4



For y-int.
 $y = \frac{1}{0+2} + 4$
 $= 4\frac{1}{2}$ or 4.5

Domain $(-\infty, -2) \cup (-2, \infty)$ Range $(-\infty, 4) \cup (4, \infty)$

23) $f(x) = -\frac{1}{x-3} - 5$

x-Int. 2.8

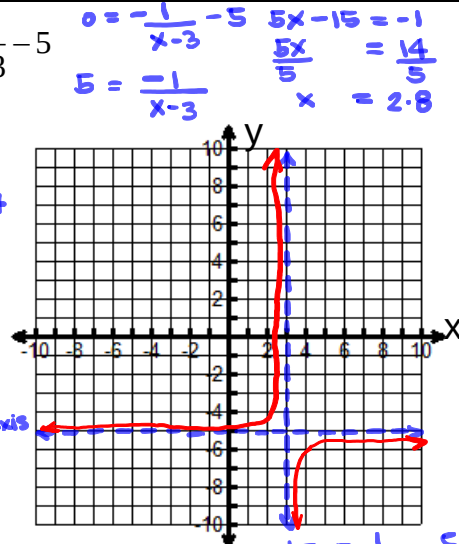
y-int. $4\frac{2}{3}$ or 4.67

V.A. $x = 3$

H.A. $y = -5$

Transformations:

- Reflection over x-axis
- Right by 3
- Down by 5



$y = -\frac{1}{0-3} - 5$
 $y = \frac{1}{3} - 5$
 $= 4\frac{2}{3}$

Domain $(-\infty, 3) \cup (3, \infty)$ Range $(-\infty, -5) \cup (-5, \infty)$

24) $f(x) = \frac{3}{x} - 2$

x-Int. $\frac{3}{2}$

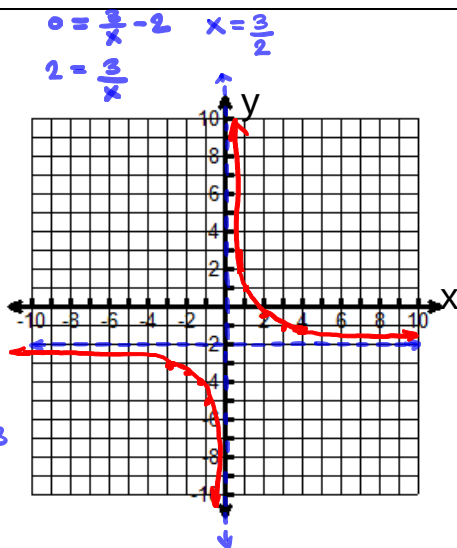
y-int. none

V.A. $x=0$

H.A. $y=-2$

Transformations:

- Vertical stretch by 3
- Down by 2



Domain $(-\infty, 0)(0, \infty)$ Range $(-\infty, -2)(-2, \infty)$

25) $f(x) = -\frac{4}{x-2}$

x-Int. none ($y \neq 0$)

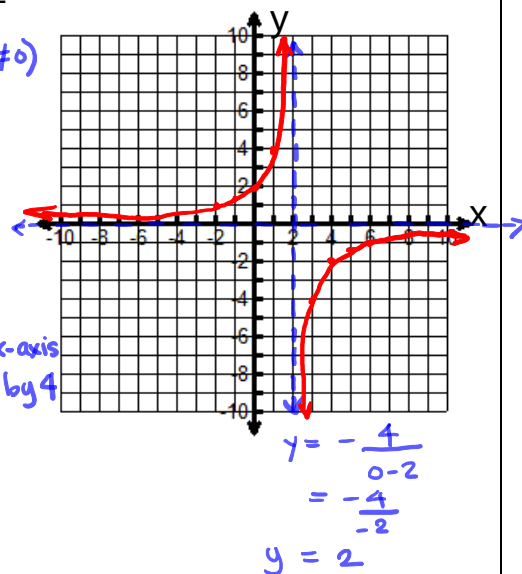
y-int. 2

V.A. $x=2$

H.A. $y=0$

Transformations:

- Reflection over x-axis
- Vertical stretch by 4
- Right by 2



Domain $(-\infty, 2)(2, \infty)$ Range $(-\infty, 0)(0, \infty)$

26) $f(x) = \frac{2}{x+5} + 2$

$0 = \frac{2}{x+5} + 2 \quad -2 = \frac{2}{x+5} \quad -2(x+5) = 2 \quad -2x - 10 = 2 \quad -2x = 12 \quad x = -6$
 $-2 = \frac{2}{x+5} \quad x = -6$

$y = \frac{2}{0+5} + 2 = \frac{2}{5} + 2 = 2\frac{2}{5}$

x-Int. -6

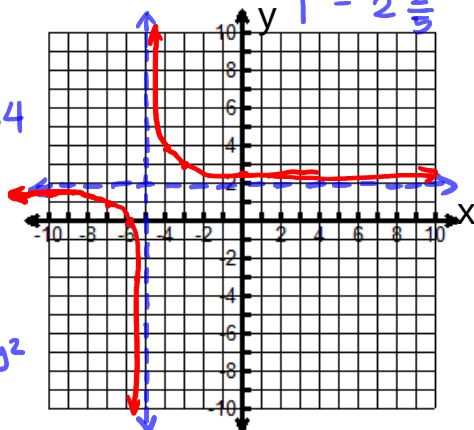
y-int. $2\frac{2}{5}$ or 2.4

V.A. $x=-5$

H.A. $y=2$

Transformations:

- vertical stretch by 2
- Left by 5
- Up by 2



Domain $(-\infty, -5)(-5, \infty)$ Range $(-\infty, 2)(2, \infty)$

27) $f(x) = \frac{1}{-2x+4} - 3 = \frac{1}{-2(x-2)} - 3$

x-Int. $\frac{11}{6}$

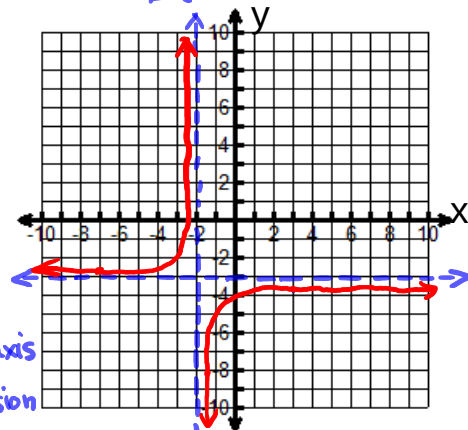
y-int. $-2\frac{3}{4}$

V.A. $x=-2$

H.A. $y=-3$

Transformations:

- Reflection over y-axis
- Horizontal compression by $\frac{1}{2}$
- Right by 2
- Down by 3
- or - Reflection over x-axis
- Vertical compression by $\frac{1}{2}$



Domain $(-\infty, -2)(-2, \infty)$ Range $(-\infty, -3)(-3, \infty)$

$0 = \frac{1}{-2(x-2)} - 3$
 $-2(x-2) \left[3 = \frac{1}{-2(x-2)} \right] - 2(x-2)$
 $-6x + 12 = 1$
 $x = \frac{-11}{-6} \text{ or } \frac{11}{6}$

$y = \frac{1}{-2(0-2)} - 3$
 $= \frac{1}{4} - 3$
 $= -2\frac{3}{4}$