Name: Key Period:____

Review Test 1: Quadratic Graphs and Their Attributes (Features)

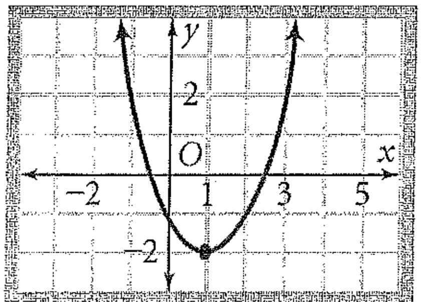
Graphing quadratic equations in standard form, vertex form, and intercept form.

A. Intro to Graphs of Quadratic Equations: $y = ax^2 + bx + c$

- A Quadratic function is a function that can be written in the form $y = ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$. Ex: $y = 5x^2$ $y = -2x^2 + 7$ $y = x^2 - x - 3$
- The graph of a quadratic function is a U-shaped curve called a Parabola. The maximum or minimum point is called the Vertex

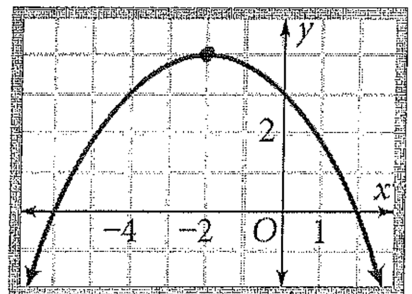
<u>Identify the vertex of each graph; identify whether it is a minimum or a maximum.</u>

1.)



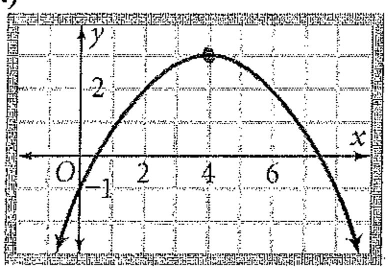
Vertex: (1 , -2) Minimum

2.)



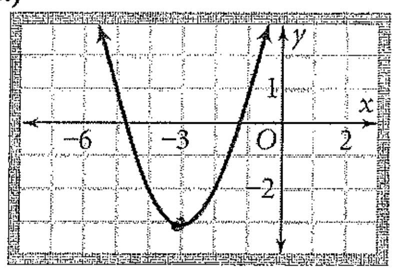
Vertex: (-2 , 4) Maximum

3.)



Vertex: (4 , 3) Maximum

4.)



Vertex: (-3 , -3) Minimum

<u>B. Key Features of a Parabola:</u> $y = ax^2 + bx + c$

- **Direction of Opening:** When $a > 0$, the parabola opens Upward →
  When $a < 0$, the parabola opens Downward →
- **Width:** When $|a| < 1$, the parabola is Wider than $y = x^2$
  When $|a| = 1$, the parabola is the Same width as $y = x^2$
  When $|a| > 1$, the parabola is Narrower than $y = x^2$
- **Vertex:** The highest or lowest point of the parabola is called the vertex, which is on the axis of symmetry. To find the vertex, plug in $x = \frac{-b}{2a}$ and solve for *y*. This yields a point (x , y)
- **Axis of symmetry:** This is a vertical line passing through the vertex. Its equation is: $x = \frac{-b}{2a}$
- ***x*-intercepts:** are the 0, 1, or 2 points where the parabola crosses the *x*-axis. Plug in $y = 0$ and solve for *x*.
- ***y*-intercept:** is the point where the parabola crosses the *y*-axis. Plug in $x = 0$ and solve for *y*.

Without graphing the quadratic functions, complete the requested information:

5.) $f(x) = 3x^2 - 7x + 1$

What is the direction of opening? Upward
Is the vertex a max or min? Min
Wider or narrower than $y = x^2$? narrower
$3 > 1$

6.) $g(x) = -\frac{5}{4}x^2 + x - 3$

What is the direction of opening? downward
Is the vertex a max or min? Max
Wider or narrower than $y = x^2$? narrower $\frac{5}{4} > 1$

7.) $y = \frac{2}{3}x^2 - 11$

What is the direction of opening? Upward
Is the vertex a max or min? Min
Wider or narrower than $y = x^2$? Wider $\frac{2}{3} < 1$

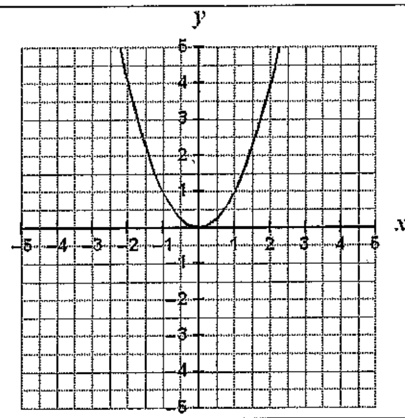
8.) $y = -0.6x^2 + 4.3x - 9.1$

What is the direction of opening? downward
Is the vertex a max or min? Max
Wider or narrower than $y = x^2$? wider $0.6 < 1$

The parabola $y = x^2$ is graphed to the right.

Note its vertex (0 , 0) and its width.

You will be asked to compare other parabolas to this graph.

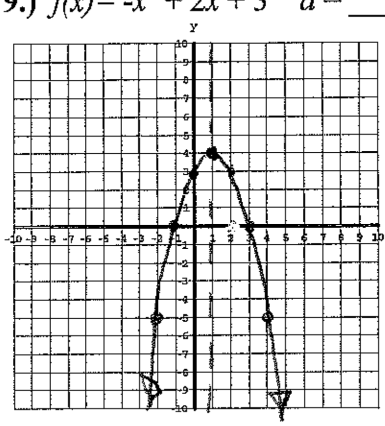


C. Graphing in STANDARD FORM ($y = ax^2 + bx + c$): we need to find the vertex first.

Vertex	**Graphing**
- list $a =$ ____, $b =$ ____, $c =$ ____ - find $x = \frac{-b}{2a}$ - plug this x-value into the function (table) - this point (___, ___) is the vertex of the parabola	- put the vertex you found in the center of your x-y chart. - choose 2 x-values less than and 2 x-values more than your vertex. - plug in these x values to get 4 more points. - graph all 5 points

Find the vertex of each parabola. Graph the function and find the requested information

9.) $f(x) = -x^2 + 2x + 3$ $a =$ -1, $b =$ 2, $c =$ 3



vertex (x, y)

AOS equation $x = \frac{-b}{2a} = \frac{-(2)}{2(-1)} = 1$

$y = -(1)^2 + 2(1) + 3$
$= -1 + 2 + 3 = 4$

x	y
-1	0
1	4
-2	-5

Reflected on AOS

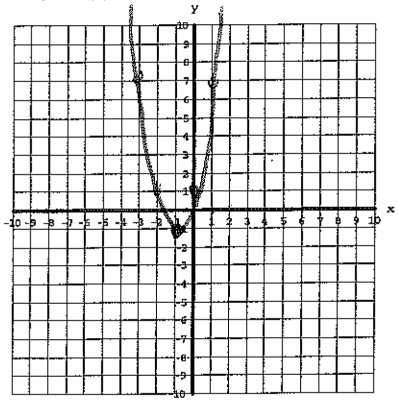
x	y
3	0
1	4
4	-5

Vertex: (1, 4)
Max or min? Max
Direction of opening? downward
Axis of symmetry: $x = 1$
Compare to the graph of $y = x^2$
- facing down
- Horizontal 1 unit right
- Vertical Shift 4 unit up

10.) $h(x) = 2x^2 + 4x + 1$

$a=2$, $b=4$, $c=1$ — AOS equation: $x = \frac{-b}{2a} = \frac{-(4)}{2(2)} = -1$

x	y
-2	1
-1	-1
-3	7



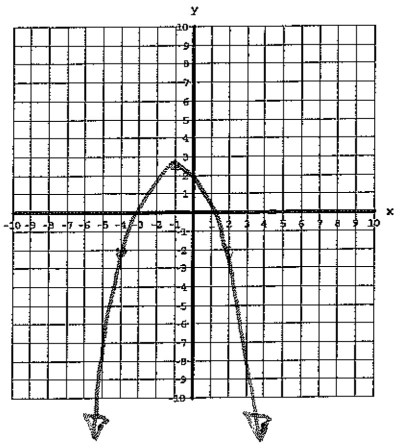
$y = 2(-1)^2 + 4(-1) + 1$
$= 2 - 4 + 1 = -1$

Vertex $(x, y) = (-1, -1)$

Vertex: $(x, y) = (-1, -1)$
Max or min? Min
Direction of opening? upward
Axis of symmetry: $x = -1$
Compare to the graph of $y = x^2$
- Horizontal shift 1 left
- Vertical shift 1 down
- Vertical stretch by 2

11.) $k(x) = 2 - x - \frac{1}{2}x^2 \rightarrow y = -\frac{1}{2}x^2 - x + 2$

$a = -\frac{1}{2}$, $b = -1$, $c = 2$ — AOS equation: $x = \frac{-b}{2a} = \frac{-(-1)}{2(-\frac{1}{2})} = -1$



$y = -\frac{1}{2}(-1)^2 - (-1) + 2 =$

$y = -\frac{1}{2} + 1 + 2 = 2.5$

2	-2
-1	2.5
-4	-2

Vertex: $(-1, 2.5)$
Max or min? Max
Direction of opening? downward
Axis of symmetry: $x = -1$
Compare to the graph of $y = x^2$
- Vertical shift 2.5 up.
- facing down
- wider
- ~~Shifted~~ Horizontal shift one left

12.) State whether the function $y = -3x^2 + 12x - 6$ has a minimum value or a maximum value. Then find the minimum or maximum value. is a Maximum

Maximum value is the y coordinate value of the vertex.

$x = \frac{-b}{2a} = \frac{-(12)}{2(-3)} = 2$

$y = -3(2)^2 + 12(2) - 6$
$= -12 + 24 - 6$
$= 6$ Maximum value $= 6$.

13.) Find the vertex of $y = \frac{1}{2}x^2 + 5x - 7$. State whether it is a minimum or maximum. Find that minimum or maximum value.

$a = \frac{1}{2}$, $b = 5$, $c = -7$

$x = \frac{-b}{2a} = \frac{-(5)}{2(\frac{1}{2})} = -5$

$y = \frac{1}{2}(-5)^2 + 5(-5) - 7$
$= \frac{1}{2}(25) - 25 - 7$
$= 12.5 - 32$
$= -29.5$

Vertex $(-5, -29.5)$

Minimum value $= -29.5$

Another useful form of the quadratic function is the vertex form: $y = a(x-h)^2 + k$.

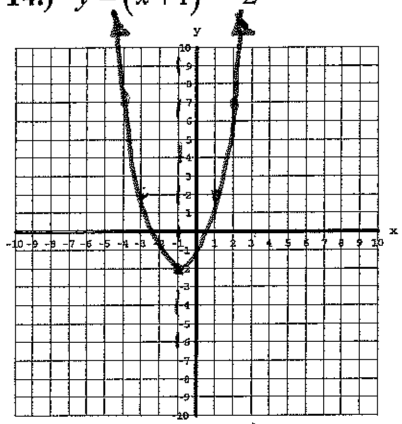
GRAPH OF VERTEX FORM $y = a(x-h)^2 + k$

The graph of $y = a(x-h)^2 + k$ is the parabola $y = ax^2$ translated Left/Right h units and Up/Down k units.

- The vertex is (h , k).
- The axis of symmetry is $x = \frac{-b}{2a}$
- The graph opens up if a $>$ 0 and down if a $\leq$ 0.

Find the vertex of each parabola and graph.

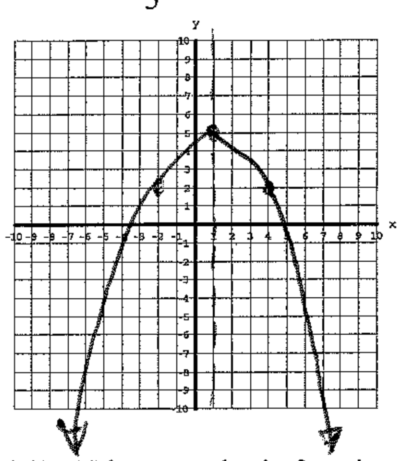
14.) $y = (x+1)^2 - 2$



x	y
2	7
1	2
-1	-2
-3	2
-4	7

Vertex: $(-1, -2)$

15.) $y = -\frac{1}{3}(x-1)^2 + 5$



x	y
4	2
1	5
-2	2

Vertex: $(1, 5)$

16.) Write a quadratic function in vertex form for the function whose graph has its vertex at (-5, 4) and passes through the point (7, 1).

$y = a(x-(-5))^2 + 4$

$y = a(x+5)^2 + 4$

$1 = a(7+5)^2 + 4$

$1 = a(12)^2 + 4$

$1 = 144a + 4$

$-4 \qquad -4$

$-3 = 144a$

$-3 = 144a$

$\frac{-3}{144} = a$

$a = \frac{-1}{48}$

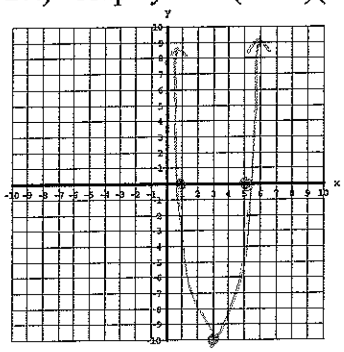
$y = \frac{-1}{48}(x+5)^2 + 4$

GRAPH OF INTERCEPT FORM $y = a(x-p)(x-q)$:

Characteristics of the graph $y = a(x-p)(x-q)$:

- The x-intercepts are p and q.
- The axis of symmetry is halfway between (p, 0) and (q, 0) and it has equation $x = \frac{\boxed{p+q}}{2}$
- The graph opens up if a > 0 and opens down if a < 0.

17.) Graph $y = -2(x-1)(x-5)$



$\frac{1+5}{2} = 3$

$x=3$

$-2(0-1)(0-5)$

$-2(-1)(-5)$

-10

use calculator

x-intercepts: (1, 0), (5, 0)

Vertex: (3, −10)

Converting between forms:

From intercept form to standard form • Use FOIL to multiply the binomials together • Distribute the coefficient to all 3 terms	Ex: $y = -2(x+5)(x-8)$ * $y = -2(x^2+5x-8x-40)$ $y = -2(x^2-3x-40)$ $y = -2x^2+6x+80$
From vertex form to standard form • Re-write the squared term as the product of two binomials • Use FOIL to multiply the binomials together • Distribute the coefficient to all 3 terms • Add constant at the end	Ex: $f(x) = 4(x-1)^2 + 9$ * $f(x) = 4(x-1)(x-1)+9$ $f(x) = 4(x^2-2x+1)+9$ $f(x) = 4x^2-8x+4+9$ $f(x) = 4x^2-8x+13$

Write Quadratic Functions and Models

A. When given the vertex and a point

- **Plug the vertex in for (h, k) in** $y = a(x-h)^2 + k$
- **Plug in the given point for (x, y)**
- **Solve for a. Plug in a, h, k into** $y = a(x-h)^2 + k$

$y = a(x-h)^2 + k$

6

18.) Write a quadratic equation in vertex form for the parabola shown.

vertex: $(\overset{h}{-2}, \overset{k}{-3})$ pt: $(\overset{x}{0}, \overset{y}{5})$

$5 = a(0-(-2))^2 \cancel{+k} - 3$

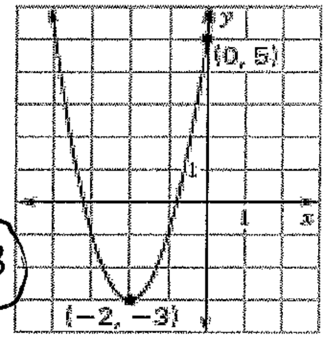
$5 = a(2)^2 - 3$

$5 = 4a - 3$
$+3 \qquad +3$

$8 = 4a$

$a = 2$

$\boxed{y = 2(x+2)^2 - 3}$



19.) Write a quadratic function in vertex form for the function whose graph has its vertex at (2, 1) and passes through the point (4, 5).

vertex: $(\overset{h}{2}, \overset{k}{1})$ pt: $(\overset{x}{4}, \overset{y}{5})$

$y = a(x-h)^2 + k$

$5 = a(4-2)^2 + 1$

$5 = a(2)^2 + 1$

$5 = 4a + 1$
$-1 \qquad -1$

$4 = 4a$

$a = 1$

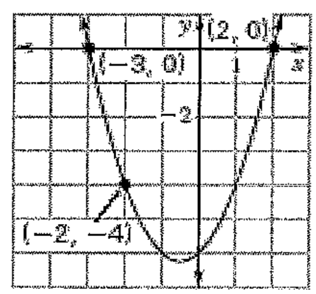
$\boxed{y = (x-2)^2 + 1}$

B. When given the x-intercepts and a third point

- **Plug in the x-intercepts as *p* and *q* into $y = a(x-p)(x-q)$**
- **Plug in the given point for (x, y)**
- **Solve for *a*. Plug in *a, h, k* into $y = a(x-p)(x-q)$**

*don't do yet (next test)

20.) Write a quadratic function in intercept form for the parabola shown.



B. When given three points on the parabola

- **Label all three points as (x, y)**
- **Separately, plug in each point into $y = ax^2 + bx + c$**
- **You now have 3 equations with three variables: a, b, c**
- **Solve for a, b, and c using elimination. Plug back into $y = ax^2 + bx + c$**

21.) Write a quadratic function in standard form for the parabola that passes through the points (–2, –6), (0, 6) and (2, 2).

22.) Write a quadratic function in standard form for the parabola that passes through the points (–1, –2), (1, –4) and (2, 1).

* dont do yet next test *

Skip

To graph a quadratic function, you must FIRST find the vertex (h, k)!!

(A) If the function starts in standard form $y = ax^2 + bx + c$:

1st: The x-coordinate of the vertex, h, = $\dfrac{-b}{2a}$

2nd: Find the y-coordinate of the vertex, k, by plugging the x-coordinate into the function & solving for y.

(B) If the function starts in intercept form $y = a(x-p)(x-q)$:

1st: Find the x-intercepts by setting the factors with x equal to 0 & solving for x.
2nd: The x-coordinate of the vertex is half way between the x-intercepts.
3rd: Find the y-coordinate of the vertex, k, by plugging the x-coordinate into the function & solving for y.

(C) If the function starts in vertex form $y = a(x-h)^2 + k$:

1st: pick out the x-coordinate of the vertex, h. REMEMBER: h will have the OPPOSITE sign as what is in the parenthesis!!
2nd: Pick out the y-coordinate of the vertex, k. It will have the SAME sign as the what is in the equation!

AFTER finding the vertex:

Make a table of values with **5 points**: The vertex, plug in 2 x-coordinates SMALLER than the x-coordinate of the vertex & 2 x-coordinates LARGER than the x-coordinate of the vertex.

Direction of Opening:

If a is positive, the graph opens up
If a is negative, the graph opens down.

Width of the function:

If $|a| > 1$, the graph is narrower than $y = x^2$

If $|a| < 1$, the graph is wider than $y = x^2$

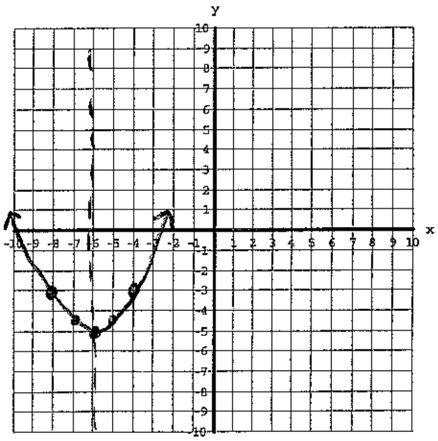
Stretch compress → Stretch

-1 0 1

Graph each function by making a table of values with at least 5 points. (A) State the vertex. (B) State the direction of opening (up/down). (C) State whether the graph is wider, narrower, or the same width as $y = x^2$.

23.) $f(x) = \frac{1}{2}(x+6)^2 - 5$ h k

vertex: $(-6, -5)$ h k



x	y
-8	-3
-7	-4.5
-6	-5
-5	-4.5
-4	-3

vertex

Vertex: $(-6, -5)$

Direction of opening? up

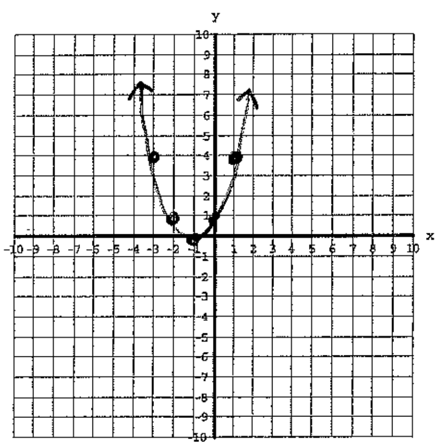
Compare width to the graph of $y = x^2$ wider (compression)

24.) $k(x) = x^2 + 2x + 1$

$a = 1$ $b = 2$ $c = 1$

$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2}$ AoS → $x = -1$

x	y
-3	4
-2	1
-1	0
0	1
1	4



$y = (-1)^2 + 2(-1) + 1$

$y = 1 - 2 + 1$

$y = 0$

Vertex: $(-1, 0)$

Direction of opening? up

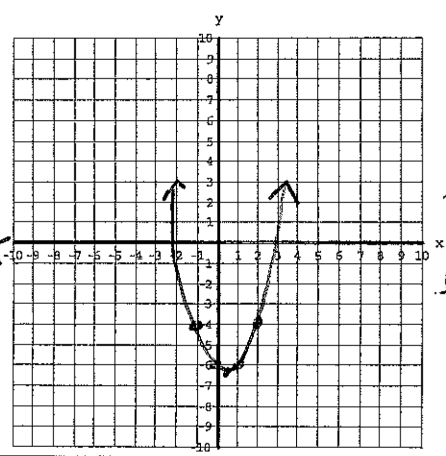
Compare width to the graph of $y = x^2$ Same

25.) $f(x) = x^2 - x - 6$

$a = 1$ $b = -1$ $c = -6$

$x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$

x	y
-1	-4
0	-6
$\frac{1}{2}$	-6.25
1	-6
2	-4



$f(x) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 6$

$f(x) = \frac{1}{4} - \frac{1}{2} - 6$

$= \frac{-25}{4} \approx -6.25$

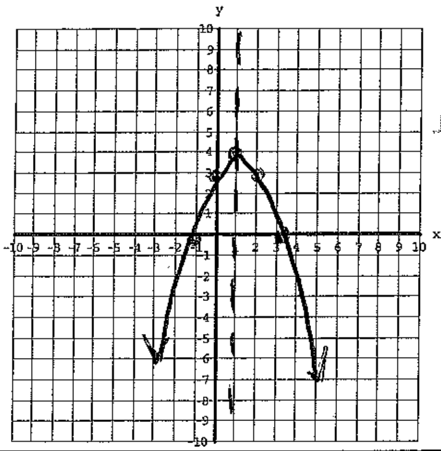
Vertex: $\left(\frac{1}{2}, \frac{-25}{4}\right)$

Direction of opening? up

Compare width to the graph of $y = x^2$ Same

26.) $f(x)=-x^2+2x+3$ $a=-1 \quad b=2 \quad c=3$

$x=\frac{-b}{2a}=\frac{-2}{2(-1)}=\frac{-2}{-2}=1$ AOS $x=1$



$f(x)=-(1)^2+2(1)+3$

$=-1+2+3$

$=1+3=4$

Vertex: $(1,4)$

Vertex: $(1,4)$

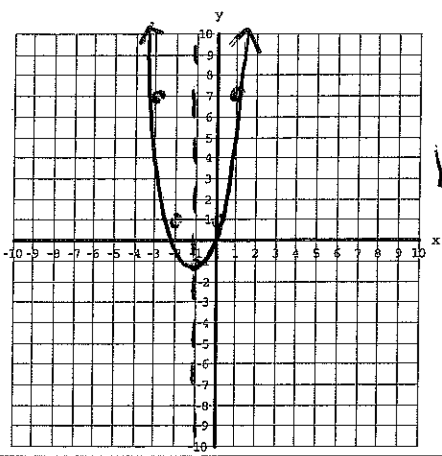
Direction of opening? down

Compare width to the graph of $y=x^2$ same

x	y
-1	0
0	3
1	4
2	3
3	0

27.) $h(x)=2x^2+4x+1$ $a=2 \quad b=4 \quad c=1$

$x=\frac{-b}{2a}=\frac{-4}{2(2)}=\frac{-4}{4}=-1$ AOS: $x=-1$



$h(x)=2(-1)^2+4(-1)+1$

$=2(1)-4+1$

$=-2+1$

$=-1$

Vertex: $(-1,-1)$

Direction of opening? up

Compare width to the graph of $y=x^2$ narrower (stretch)

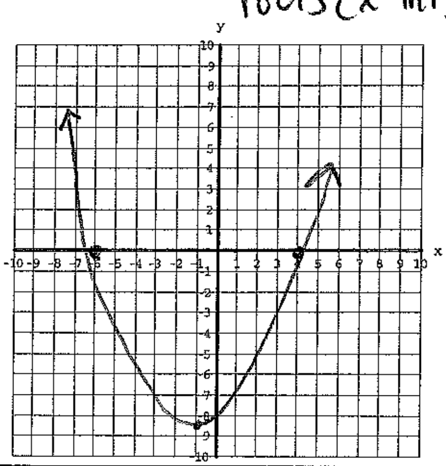
vertex: $(-1,-1)$

x	y
-3	7
-2	1
-1	-1
0	1
1	7

28.) $g(x)=-\frac{1}{3}(x-4)(x+6)$ *use calculator.

$x=4 \quad x=6$

roots (x-int) $x=\frac{4+6}{2}=\frac{10}{2}\ \frac{-2}{2}=-1$



$y=-\frac{1}{3}(-1-4)(-1+6)$

$y=-\frac{1}{3}(-5)(+5)$

$y=\frac{+25}{3}=8.\overline{3}$

Vertex: $(-1,8.\overline{3})$

Direction of opening? down

Compare width to the graph of $y=x^2$ wider (compression) a b c

29. The vertex of the parabola represented by $y = 2x^2 - 4x - 4$ is:

a) (1, -6) b) (1, -5) ~~c)~~ (-1, 2) ~~d)~~ (-1, 6)

$x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$

$y = 2(1)^2 - 4(1) - 4$

$y = 2 - 4 - 4 = -6$

vertex: (1, -6)

30. The axis of symmetry for the parabola defined by $y = x^2 - 8x + 12$:

a b c

a) x = -4 b) x = 4 c) x = 8 d) x = 12

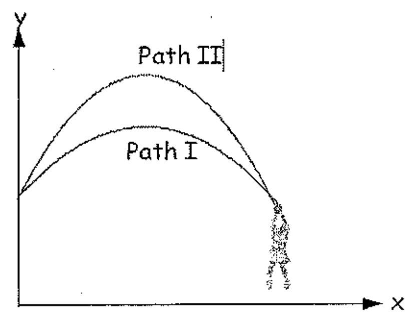
$x = \frac{-b}{2a} = \frac{-(-8)}{2(1)} = \frac{8}{2} = 4$ AOS: $x = 4$

31. A cannon ball is shot into the air and its height in meters is represented by $h = 1.5 + 23.1t - 4.9t^2$ where "t" is time in seconds. How high does the cannon ball go? * use calc. *

a) 23.10 m b) 4.78 m c) 28.73 m d) 2.36 m (Max value)

32. A basketball player is trying to increase her shot accuracy. She stays in the same position on the court and increases the arc of the flight path of the ball.

Her coach graphs the quadratic function $\frac{1}{a}(y-k) = (x-h)^2$ to model parabolic Path I of the basketball. The coach then Changes certain values in the given Equation to graph path II. Which value(s) Did the coach **NOT** change?



a) a b) h
c) k d) h and a

$y = a(x-h)^2 + k$

does not move left or right