CALCULUS AB ASSIGNMENT SHEET

UNIT 6

NOV 7 - NOV 18

Mon, Nov 7	3.6 Curve Sketching given f, f' and f''	Packet p. 3 # 1-6
Tues, Nov 8	Review Game	
Wed, Nov 9	Test over 3.1 – 3.4	
Thur., Nov 10	3.5 Limits at Infinity and Sketching Rational Functions	p. 202 # 17-33 odd,
Fri., Nov 11	AP Derivative Graphs	Notes on Derivative Graphs
Mon, Nov 14	**Quiz over curve sketching and limits**	
Tues., Nov 15	**Derivative Graphs Day 2**	
Wed, Nov 16	Quiz More Derivative Graphs	
Thur, Nov 17	Review Game	Study for Test
Fri, Nov 18	**Test** over 3.4, 3.5, 3.6, 3.7	

SECTION 3.6

SKETCHING THE GRAPH

Example 1 $f(x) = x^4 - 12x^3 + 48x^2 - 64x$ $f'(x) = 4(x-1)(x-4)^2$ $f''(x) = 12(x-4)(x-2)$

Example 2 $f(x) = 2x^{5/3} - 5x^{4/3}$ $f'(x) = \frac{10}{3}x^{1/3}(x^{1/3} - 2)$ $f''(x) = \frac{20(x^{1/3} - 1)}{9x^{2/3}}$

Example 3 $f(x) = \frac{x}{\sqrt{x^2 + 2}}$ $f'(x) = \frac{2}{(x^2 + 2)^{3/2}}$ $f''(x) = \frac{-6x}{(x^2 + 2)^{5/2}}$

Assignment

1) $f(x) = \frac{x^2}{x^2 + 3}$ $f'(x) = \frac{6x}{(x^2 + 3)^2}$ $f''(x) = \frac{18(1 - x^2)}{(x^2 + 3)^3}$

2) $f(x) = \frac{1}{x - 2} - 3$ $f'(x) = \frac{-1}{(x - 2)^2}$ $f''(x) = \frac{2}{(x - 2)^3}$

3) $f(x) = \frac{x^2 + 1}{x}$ $f''(x) = 1 - \frac{1}{x^2}$ $f''(x) = \frac{2}{x^3}$

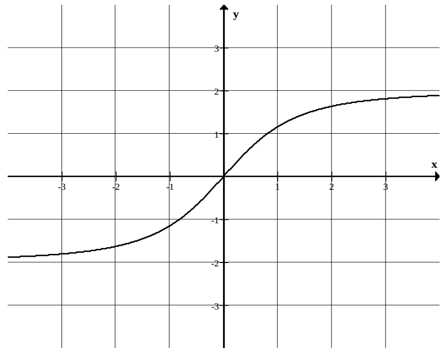
4) $f(x) = x\sqrt{4 - x}$ $f''(x) = \frac{8 - 3x}{2\sqrt{4 - x}}$ $f''(x) = \frac{3x - 16}{4(4 - x)^{3/2}}$

5) $f(x) = 3x^{2/3} - 2x$ $f'(x) = \frac{2(1 - x^{1/3})}{x^{1/3}}$ $f''(x) = \frac{-2}{3x^{4/3}}$

6) $f(x) = x^3 - 3x^2 - 3$ $f'(x) = 3x^2 - 6x$ $f''(x) = 6x - 6$

3.5 - Limits at Infinity and Horizontal Asymptotes

1. Describe the end behavior of f:



Let $f(x)$ be a rational function such that $f(x)=\dfrac{g(x)}{h(x)}$.

1. If the degree of $g(x)<h(x)$, then $\lim\limits_{x\to\infty} f(x)=0$.
2. If the degree of $g(x)=h(x)$, then $\lim\limits_{x\to\infty} f(x)=$ the ratio of the two leading coefficients.
3. If the degree of $g(x)>h(x)$, then $\lim\limits_{x\to\infty} f(x)$ does not exist.

1. Find the limits:

a.) $\lim\limits_{x\to\infty}\dfrac{3x-7}{2x^2+1}$ **b.)** $\lim\limits_{x\to\infty}\dfrac{-5x^2+7}{2x^2+3x-1}$ **c.)** $\lim\limits_{x\to\infty}\dfrac{x^3-1}{x^2+7x+10}$

Practice on Limits

<u>3.5 Limits at Infinity</u>

<u>Ex.</u> Evaluate the following limits. Hint: Divide numerator and denominator by the highest power of x in the denominator.

(a) $\lim\limits_{x\to\infty}\dfrac{3x-1}{2x+5}=$

(b) $\lim\limits_{x\to-\infty}\dfrac{3x-1}{2x+5}=$

(c) $\lim\limits_{x\to\infty}\dfrac{3x^2-1}{2x+5}=$

(d) $\lim_{x\to-\infty} \frac{3x^2-1}{2x+5} =$

(e) $\lim_{x\to\infty} \frac{3x-1}{2x^2+5} =$

(f) $\lim_{x\to-\infty} \frac{3x-1}{2x^2+5} =$

(g) $\lim_{x\to\infty} \frac{2x}{\sqrt{9x^2+4}} =$

(h) $\lim_{x\to-\infty} \frac{2x}{\sqrt{9x^2+4}} =$

(i) $\lim_{x\to\infty} \frac{\sin(5x)}{x^2} =$

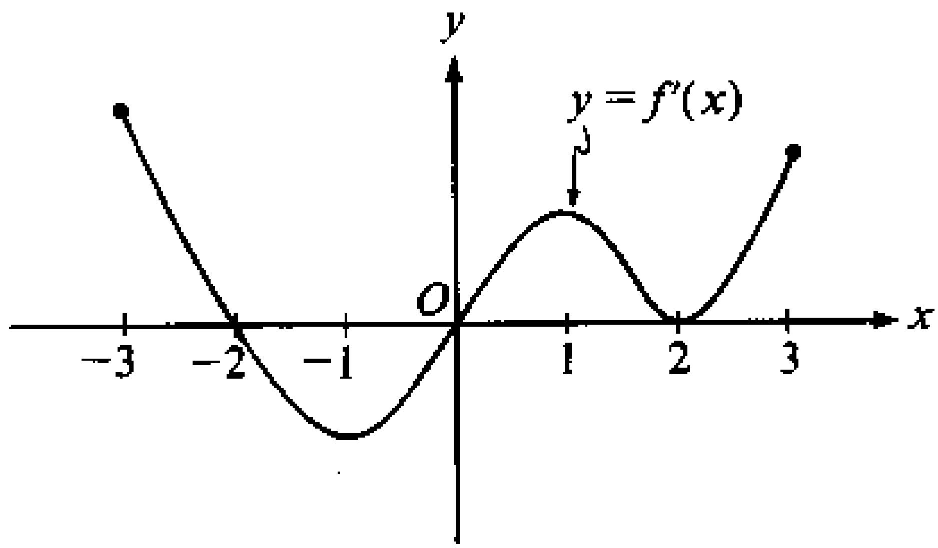
Rational Functions

Sketch a graph of each rational function.

1) $y = \frac{2+x}{1-x}$ 2) $y = \frac{x}{x^2-4}$ 3) $y = \frac{2x}{1-x}$ 4) $y = \frac{x^2}{x^2-9}$ 5) $y = \frac{2x^2}{x^2+4}$

6) $y = \frac{2x}{1-x^2}$ 7) $y = \frac{x-3}{x-2}$ 8) $y = \frac{x^2}{x^2+9}$

DERIVATIVE GRAPHS FIRST ROUND



1) Critical Numbers

2) Increasing

3) Decreasing

4) Relative Minimum

5) Relative Maximum

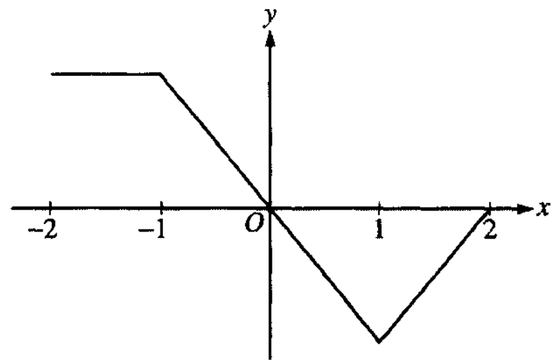
6) Inflection Point

7) Concave up

8) Concave down

9) Sketch the graph

2003



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true?

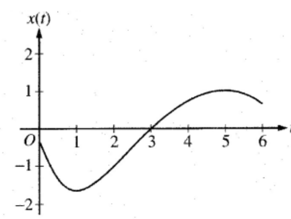
(A) f is decreasing for $-1 \le x \le 1$.

(B) f is increasing for $-2 \le x \le 0$.

(C) f is increasing for $1 \le x \le 2$.

(D) f has a local minimum at $x = 0$.

(E) f is not differentiable at $x = -1$ and $x = 1$.



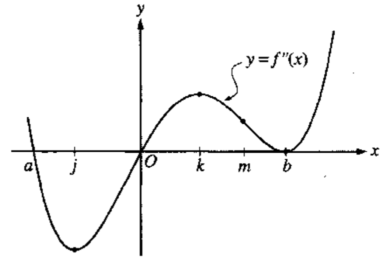
21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$.

The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$.

For what values of t is the velocity of the particle increasing?

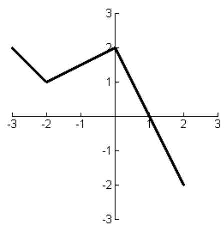
(A) $0 < t < 2$ (B) $1 < t < 5$ (C) $2 < t < 6$

(D) $3 < t < 5$ only (E) $1 < t < 2$ and $5 < t < 6$



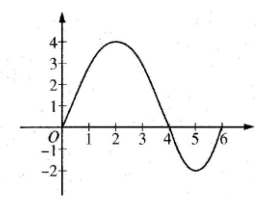
21. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only (B) 0 and m only (C) b and j only (D) 0, a, and b (E) b, j, and k



9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$

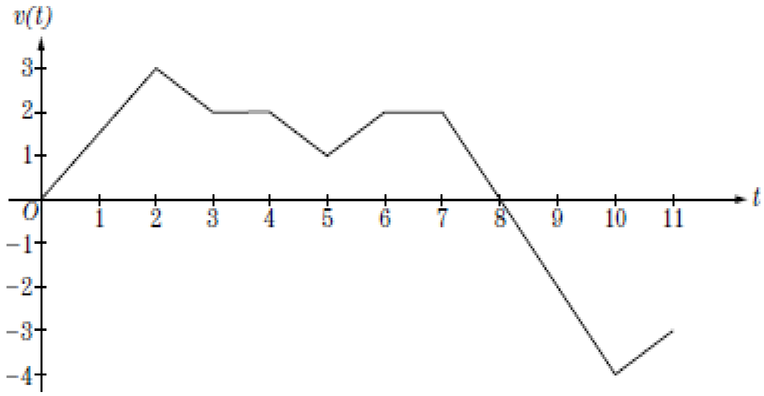


17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by

$g(x) = \int_{0}^{x} f(t)\,dt$. For what values of x does the graph of g have appoint of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

Questions 9 and 10 refer to the following graph and information.



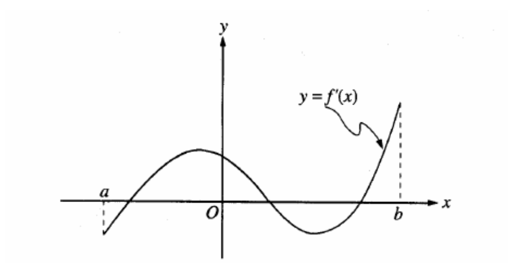
A bug is crawling along a straight wire. The velocity, $v(t)$, of the bug at time t, $0 \le t \le 11$, is given in the graph above.

9. According to the graph, at what time t does the bug change direction?

(A) 2
(B) 5
(C) 6
(D) 8
(E) 10

10. According to the graph, at what time t is the speed of the bug greatest?

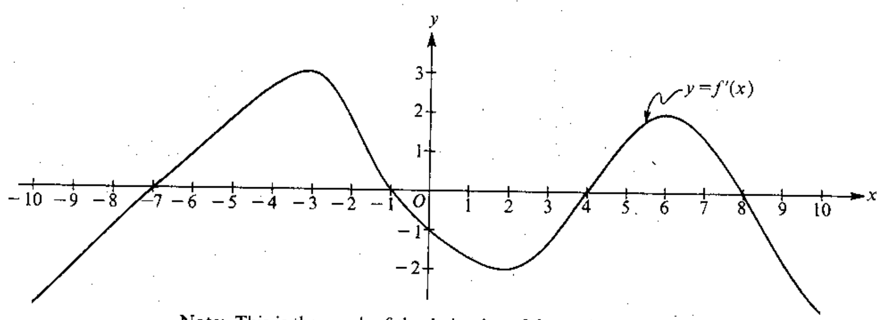
(A) 2
(B) 5
(C) 6
(D) 8
(E) 10



12. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

1989 AB 5



Note: This is the graph of the derivative of f, not the graph of f.

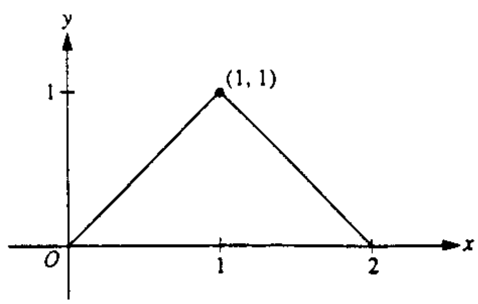
The figure above shows the graph of f', the derivative of a function *f*. The domain of *f* is the set of all real numbers *x* such that $-10 \le x \le 10$.

(a) For what values of *x* does the graph of *f* have a horizontal tangent?

(b) For what values of *x* in the interval $(-10, 10)$ does *f* have a relative maximum? Justify your answer.

(c) For what values of *x* is the graph of *f* concave downward?

1993 AB 5



Note: This is the graph of the derivative of *f*, not the graph of *f*.

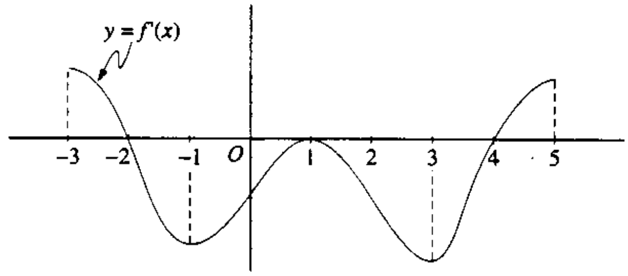
1. The figure above shows the graph of f', the derivative of a function *f*. The domain of *f* is the set of all *x* such that $0 < x < 2$.

   (a) Write an expression for $f'(x)$ in terms of *x*.

   (b) Given that $f(1) = 0$, write an expression for $f(x)$ in terms of *x*.

   (c) Sketch the graph of $y = f(x)$.

1996 AB 1



Note: This is the graph of f', not the graph of *f*.

The figure above shows the graph of f', the derivative of a function *f*. The domain of *f* is the set of all real numbers *x* such that $-3 < x < 5$.

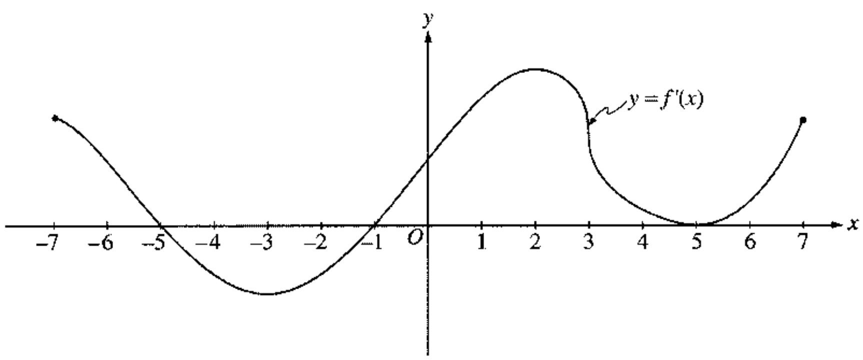
(a) For what values of *x* does *f* have a relative maximum? Why?

(b) For what values of *x* does *f* have a relative minimum? Why?

(c) On what intervals is the graph of *f* concave upward? Use f' to justify your answer.

(d) Suppose that $f(1) = 0$. Draw a sketch that shows the general shape of the graph of the function *f* on the open interval $0 < x < 2$.

2000 AB 3



The figure above shows the graph of f', the derivative of the function *f*, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, *x* = 2, and *x* = 5, and a vertical tangent line at *x* = 3.

(a) Find all values of *x*, for $-7 < x < 7$, at which *f* attains a relative minimum. Justify your answer.

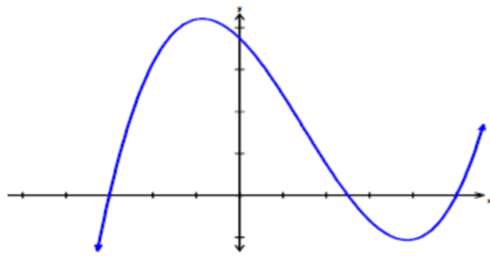
(b) Find all values of *x*, for $-7 < x < 7$, at which *f* attains a relative maximum. Justify your answer.

(c) Find all values of *x*, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what values of *x*, for $-7 \le x \le 7$, does *f* attain its absolute maximum? Justify your answer.

On the AP Calculus AB/BC exams, students are expected to analyze and interpret a variety of graphs. Given f', students should be able to answer questions about f, f' and f''. Students must be able to move between these related functions smoothly and understand what to look for in each situation. The following activity will help make these relationships clear.

Consider the graph of $f(x)$ below:



1. When is $f(x)=0$? Explain your reasoning in terms of what you see on the graph.

2. When is $f'(x)=0$? Explain your reasoning in terms of what you see on the graph.

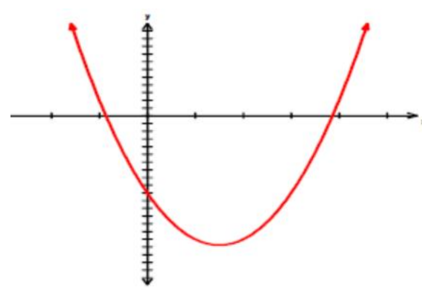
3. On what intervals is $f(x)$ increasing? Explain your reasoning in terms of the graph.

4. When does $f(x)$ have a relative minimum? Explain your reasoning using the graph.

5. When does $f''(x)=0$? Explain your reasoning using the graph.

6. When is $f''(x)<0$? Explain your reasoning using the graph.

Graph 2 is the derivative of f(x)



1. When does $f'(x)=0$? Explain your reasoning using the graph.

2. When is f(x) increasing? Explain your reasoning using the graph.

3. When does f(x)have a relative minimum? Explain your reasoning using the graph.

4. When does $f''(x)$? Explain your reasoning using the graph.

5. When is $f''(x)$<0 ? Explain your reasoning using the graph.

6. When does f(x) have a point of inflection? Explain your reasoning using the graph.

Putting It All Together

Use the word bank below to fill in the blanks for the following questions. Not all words need to be used and words can be used more than once.

zero	y-value	direction	shape	x-intercepts
positive	negative	increasing	decreasing	relative extrema

1. If given a graph of $f'(x)$, we look at ___________________ to determine the critical values for $f(x)$.

2. If given a graph of $f'(x)$, we look at __________________ to determine the possible points of inflection for $f(x)$.

3. If given a graph of $f'(x)$, we look at the __________________ to determine the slope of the tangent line to $f(x)$ at a given x-value.

4. If given a graph of $f'(x)$, $f(x)$ is concave up when $f'(x)$ is ____________________.

5. If given a graph of $f'(x)$, $f(x)$ is decreasing when $f'(x)$ is ____________________.

6. The derivative of $f(x)$ is the _________________ of $f'(x)$.

7-8. Write 2 more sentences with blanks (using the word bank above) involving f, f' and f'' analysis.

7.

8.

REVIEW FOR TEST

1. If $f(x) = 3x^4 - 16x^3 + 18x^2$, then f has a relative maximum at x =

2. What are all the values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

3. If $f(x) = \dfrac{3x+1}{2x+1}$, then which of the following is $f'(1)$?

4. If $f(x) = 4x^3 - 12x$, then f has a relative minimum at x =

5. The absolute minimum for $f(x) = 2x^3 - 3x^2 + 1$ on [- 1, 2] occurs at x =

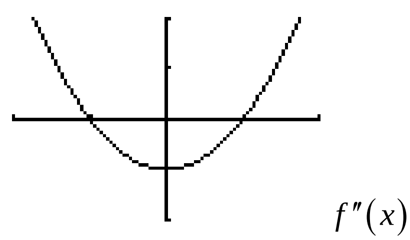
6. Which of the following are all the intervals on which the graph of $f(x) = \dfrac{x-1}{x+3}$ is concave upward?

7. What are the values of x for which the graph of $y = 8x^3 - 2x^4$ is concave downward?

8. How many points of inflection does the graph of $y = x^6 - 10x^4 + 2x - 1$ have?

9. If $f''(x) = 3x^2 - 4$ and $f(x)$ has critical numbers at -2, 0, and 2. Use the Second Derivative Test to determine at which critical number the graph of f has a local maximum?

10. Questions 10 and 11 refer to the graph of $y = f''(x)$ shown below:



10. On which intervals is the graph of f concave up?

11. On which intervals is f' decreasing?

MORE Review for test Problems below plus limits at infinity

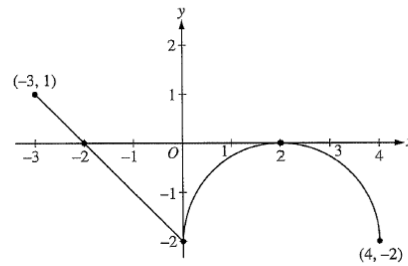
1) Given: $f(x)=\frac{4x}{\sqrt{x^2+15}}$ $f'(x)=\frac{60}{(x^2+15)^{3/2}}$ $f''(x)=\frac{-180x}{(x^2+15)^{5/2}}$

a. Find increasing,decreasing, critical numbers, inflection point, and make a sketch of the graph.
b. Find $\lim_{x\to\infty} f(x)$
c. Find $\lim_{x\to-\infty} f(x)$

2) Given: $f(x)=18(x-3)(x-1)^{2/3}$ $f'(x)=\frac{30(x-\frac{9}{5})}{(x-1)^{1/3}}$ $f''(x)=\frac{20(x-\frac{3}{5})}{(x-1)^{4/3}}$

Find critical numbers, relative extrema, inflection points and sketch.

3) $f(x) = 3x^4 - 6x^2$ How many turning points?



Graph of f'

4) Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0)=3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as show on the right.

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point (0,3).

(d) Find f ' (-3) and f ' (4) .

5) $f(x)=3x^4-8x^3+6x^2+1$ $f'(x)=12x(x-1)^2$ $f''(x)=12(x-1)(3x-1)$