**Kinematics**

**1 D Kinematics**

<table>
<thead>
<tr>
<th>x or y direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{v} = v_0 + at )</td>
</tr>
<tr>
<td>( x-x_0 = v_0 t + \frac{1}{2} at^2 )</td>
</tr>
<tr>
<td>( v^2 = v_0^2 + 2a(x-x_0) )</td>
</tr>
</tbody>
</table>

At peak height, \( v=0 \)

If starts from rest or dropped, \( v=0 \)

If displacement is positive if ends up higher or to right

Displacement is negative if ends up lower or to left

Vertical acceleration for projectile is always \(-9.8 \text{ m/s}^2\) (even on the way up)

Even with negative acceleration you can be speeding up (if velocity and acceleration have same sign)

**2 D Kinematics**

\[ y \text{ direction} \]
\[ v_y = (v_0 \sin \theta) + (-9.8)t \]
\[ y - y_0 = (v_0 \sin \theta) t + \frac{1}{2} (-9.8)t^2 \]
\[ v_y^2 = (v_0 \sin \theta)^2 + 2(-9.8)(y - y_0) \]

\[ x \text{ direction} \]
\[ d = (v_0 \cos \theta) t \]

**Graphing motion**

(Note: the graphs below do not represent the same moving object)

**For position vs. time**
- Slope is velocity
- Steep slope means moving fast
- Curvature is acceleration

**For velocity vs. time**
- Slope is acceleration
- Going towards \( v=0 \) axis means slowing
- Horiz. line means constant speed

Note: Area under \( v \) vs. \( t \) gives you displacement
**Forces**

**Newton’s Laws**

1. Objects maintain **constant velocity** unless there is force.  
   - “No forces” does not mean “no motion”. “No forces” means no acceleration (constant motion/velocity).

2. $\Sigma F = ma$
   - You need to choose only one direction at a time, e.g. $\Sigma F_x = ma_x \quad \Sigma F_y = ma_y$

3. Every force on an object, has an **equal and opposite** force on a different object.  
   - Since the forces are on different objects they do not cancel.

**Some common kinds of forces**

<table>
<thead>
<tr>
<th>Force Type</th>
<th>Formula</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{\text{gravity}}$</td>
<td>$mg = \frac{Gm_1m_2}{r^2}$</td>
<td>Note: On another planet $g = \frac{GM_p}{R^2}$</td>
</tr>
<tr>
<td>$F_{\text{normal}}$</td>
<td>$mg$</td>
<td>unless</td>
</tr>
<tr>
<td>$F_{\text{friction}}$</td>
<td>$\mu_s F_{\text{normal}}$</td>
<td></td>
</tr>
<tr>
<td>$F_{\text{kinetic}}$</td>
<td>$\mu_k F_{\text{normal}}$</td>
<td></td>
</tr>
<tr>
<td>$F_{\text{spring}}$</td>
<td>$kx$</td>
<td></td>
</tr>
<tr>
<td>$F_{\text{buoyancy}}$</td>
<td>$\rho Vg$</td>
<td></td>
</tr>
<tr>
<td>$F_{\text{electric}}$</td>
<td>$kq_1q_2/r^2 = qE$</td>
<td></td>
</tr>
<tr>
<td>$F_{\text{magnetic}}$</td>
<td>$qvBs\sin\theta = ILB\sin\theta$</td>
<td></td>
</tr>
</tbody>
</table>

Note: Scales measure normal force

1. On incline $F_n = mg \cos \theta$
2. Being pushed down or up $F_n = mg \pm fsin\theta$
3. In elevator going up or down $F_n = m(g \pm a)$
Common types of force problems

**Inclined plane**
- Friction force: $\mu Mg \cos \theta$
- Normal force: $Mg \cos \theta$
- $Mg \sin \theta$ (component that makes it slide)
- Note: if no friction $a = g \sin \theta$

**Mass pulled by string**
- $F_n = Mg - T \sin \theta$
- $\mu (Mg - T \sin \theta)$
- $T \cos \theta$
- $Mg$

**Table and pulley**
- $F_n = Mg$
- $\mu Mg$
- $a = \frac{(mg - \mu Mg)}{(M+m)}$

**Mass pushed by person**
- $F_n = Mg + F \sin \theta$
- $F$ exerted by person
- $T \cos \theta$
- $Note: F_k = \mu (Mg + F \sin \theta)$

**Mass on string**
- $T = Mg \cos \theta$ (if in equilibrium)
- $If \ equilibrium \ a = 0$
- $If \ swinging \ a = v^2/r$

**Pulley**
- $a = \frac{(Mg - mg)}{(M+m)}$
Solving 2D force (vector) problems

1. **Draw the forces** exerted on the object you are concerned with

   (These are forces ON the object, not the forces the object is exerting on other objects)

2. **Break each force into vertical and horizontal components $F_y$ and $F_x$**

   (For completely vertical or horizontal forces one component will be zero, the other is $\pm F$)

   (For diagonal forces you need to use sin and cos to break the vector into components)

   (Up or Right is a positive component, Left or Down is a negative component)

3. **Use $\Sigma F_y = ma_y$ and $\Sigma F_x = ma_x$**

   i.e. for the example shown $\Sigma F_x = F_a \cos \theta + 0 + (-F_c) = Ma_x$ \n   $\Sigma F_y = F_a \sin \theta + (-F_b) + 0 = Ma_y$

4. **If object is not moving in the x direction (or constant speed in x direction) then $\Sigma F_x = 0$**

   Similarly for the y direction, if the object is not moving vertically (or it is moving at constant speed vertically) then $\Sigma F_y = 0$
Centripetal Motion

\[ \Sigma F = m \frac{v^2}{r} \]

- Forces directed into the circle are positive and forces directed out of the circle are negative.
- You can always use this if something is going in a circle (but you don't always have to use this).
- If an object is moving with constant speed in a circle then \( v = \frac{2\pi r}{T_{\text{period}}} \).

Kepler's Laws

1. Planets follow an ellipse with Sun as one focal point.
2. All planets sweep out equal areas in equal times.
3. The value of \( R^3/T^2 \) is the same for each planet.

Derivation of Kepler's 3rd Law

1. \( \Sigma F = m \frac{v^2}{r} \)
2. \( \frac{GM_p M_s}{R^2} = M_p v^2/R \)
3. \( \frac{GM_p}{R^2} = \frac{v^2}{R} \) (cancel \( M_p \))
4. \( \frac{GM_s}{R^2} = \left( \frac{2\pi R T}{2} \right)^2 \) (Sub in formula for \( v \))
5. \( \frac{GM_s}{4\pi^2} = \frac{R^3}{T^2} \) (E doub all Rs to right side and move \( 4\pi^2 \) to left side)

Note: Kepler's Laws and the 3rd Law derivation work even for objects (moon, satellite) orbiting a planet. You just use mass of planet instead of the mass of the Sun.
Energy and Work

**Work**

\[ W = F \text{dcos} \theta \]

(Note: Area under F vs. x graph equals work.)

- Work tells you how much energy is transferred.
- If a force pushes in the direction of d (tries to speed up object), force does + work.
- If the object doesn't move d=0, or the force is perpendicular to motion, \( W = 0 \).
- The work done by gravity is \( W_g = \pm mgh \) (use + if moving down, - if moving up).
- Work-Kinetic energy \( W_{\text{total}} = \Delta KE = KE_f - KE_i \)

**Types of energy**

\[ KE = \frac{1}{2}mv^2 \]

\[ PE_{\text{gravity}} = mgh \]

(you choose where \( h = 0 \))

\[ PE_{\text{spring}} = \frac{1}{2}kx^2 \]

(x is the compression or extension)

**Thermal Energy**

\[ W = \text{Fd} \]

- \( F_{\text{friction}}d \) for friction.
- \( F_{\text{air}}d \) for air resistance.
- Or just determine how much mechanical energy was lost.

\[ Q_{\text{heat}} = mc\Delta T = mL \]

\[ PE_{\text{electric}} = qV = kq_1q_2/r \]

\[ E_{\text{capacitor}} = \frac{1}{2} CV^2 = \frac{1}{2} QV \]

\[ E_{\text{photon}} = hf \]

\[ E = mc^2 \]
Moment of and Collisions

Momentum

\[ p = mv \quad [\text{kg m/s}] \]

- Momentum is a vector and direction matters (+ if going right, - if going left)
- A situation where two objects of mass \( m \) move with speed \( v \) toward each other has no total momentum
- Momentum is always conserved

\[ \Sigma F = \Delta p / \Delta t \quad [\text{which also} = ma] \]

- This is a way to relate changes in momentum to forces

Impulse \( = \Delta p = F \Delta t \quad [\text{Ns}] \)

- Area under \( F \ vs. \ time \) graph is the Impulse (change in momentum)

Collisions

- Momentum is conserved in every type of collision (elastic and inelastic)

a. Elastic collisions

- \( KE \) is conserved
- objects must bounce off each other
- Note: For elastic collisions, \( (v_1 - v_2)_{\text{initial}} = -(v_1 - v_2)_{\text{final}} \)

b. Inelastic collisions

- \( KE \) is not conserved, \( KE \) gets lost in the collision (turns into thermal energy, etc)
- objects can stick together (perfectly inelastic) or bounce off each other

Note: To determine whether a collision is elastic or inelastic just compare the \( KE_{\text{tot}} \) before and after the collision.
- If \( KE_{\text{tot initial}} = KE_{\text{tot final}} \), then it is elastic. If not, it is inelastic.
Torque and Center of Mass

**Torque**

\[ \tau = rF \sin \theta \]

- \( r \) is from the axis to the point where the force is applied

**Equilibrium**

\[ \Sigma F = 0 \quad \Sigma \tau = 0 \]

- no acceleration and no angular acceleration (usually means at rest)
- i.e. Torque in CW direction equals torque in CCW direction

**Center of mass**

\[ CM = \frac{1}{M_{tot}} \Sigma mr = \frac{1}{M_{tot}} (m_1r_1 + m_2r_2 + ...) \]

- \( r \) is distance from arbitrary point (end of object, middle of object, etc.) to mass \( m \) or \( dm \), but if there is an obvious axis it is usually a good idea to use it

- the CM tells you the position where an object could balance

- the CM is also where you could treat all the mass as residing
Waves and Simple Harmonic Motion

Simple Harmonic oscillators
- position described by $x(t) = A \sin(\omega t)$ or $x(t) = A \cos(\omega t)$

$\omega = 2\pi/T \text{[rad/sec]}$

$T_{\text{mass on spring}} = 2\pi \left[ \frac{m}{k} \right]^{1/2}$

-Spring Period $T$ does not depend on $g$ or amplitude
- $E_{\text{total}} = \frac{1}{2} k A^2$ (for horiz. mass on spring, because at the maximum displacement $A$ the KE=0)

$T_{\text{pendulum}} = 2\pi \left[ \frac{L}{g} \right]^{1/2}$

- Pendulum Period $T$ does not depend on mass or amplitude (as long as the amplitude is small $\theta < 20^\circ$)

### Waves

\[ v = \lambda f \quad (v \text{ is wave speed, } \lambda \text{ is wavelength, } f \text{ is frequency}) \]

Note: this eqn. is true for every wave

**Transverse waves:** medium moves perpendicular to velocity of wave (e.g. ripple on pond)

**Longitudinal waves:** medium moves parallel to velocity of wave (e.g. sound waves)

### Standing waves

- **Nodes:** points that don’t move (dest. int)
- **Antinodes:** points of maximum displacement (const. int)

- Open-Open pipe is node-node
- Open-closed pipe is Node-Antinode

<table>
<thead>
<tr>
<th>$\lambda = \frac{2L}{n}$</th>
<th>node-node</th>
<th>$f = \frac{nv}{2L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 2L$</td>
<td></td>
<td>$f = \frac{v}{2L}$</td>
</tr>
<tr>
<td>$\lambda = L$</td>
<td></td>
<td>$f = \frac{v}{L}$</td>
</tr>
<tr>
<td>$\lambda = \frac{2L}{3}$</td>
<td></td>
<td>$f = \frac{3v}{2L}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda = \frac{4L}{(2n-1)}$</th>
<th>node-antinode</th>
<th>$f = \frac{(2n-1)v}{4L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 4L$</td>
<td></td>
<td>$f = \frac{v}{4L}$</td>
</tr>
<tr>
<td>$\lambda = \frac{4L}{3}$</td>
<td></td>
<td>$f = \frac{3v}{4L}$</td>
</tr>
<tr>
<td>$\lambda = \frac{4L}{5}$</td>
<td></td>
<td>$f = \frac{5v}{4L}$</td>
</tr>
</tbody>
</table>
Circuits

**Current**

\[ I = \frac{Q}{t} \quad [C/sec = \text{Amperes}] \]
- defined to be in the direction of positive charge flow (or opposite direction of e⁻)
- is directed out of the + terminal of a battery, and into the - terminal

**Resistance**

The resistance of a length \( L \) of cylinder made with resistivity \( \rho \), and cross sectional area \( A \) is,

\[ R = \rho \frac{L}{A} \quad [\text{Ohms}] \]

**Ohm’s Law**

\[ V = IR \quad (V \text{ is voltage drop across resistor, } I \text{ is current through the resistor, } R \text{ is resistance}) \]
- \( V \) is not necessarily the voltage of the battery!
- **Ohmic materials** have constant resistance (slope on \( V \) vs. \( I \)), regardless of what the current is
- **Non-Ohmic materials** change their “resistance” depending on what the current/voltage is

**Electrical Power**

\[ P = IV \quad [\text{Watts}] \]

\[ P = I^2R \]

\[ P = \frac{V^2}{R} \]

**Capacitors**

\[ C = \frac{Q}{V} \quad (C \text{ is capacitance, } Q \text{ is charge on + plate, } V \text{ is voltage across capacitor}) \]
- Capacitance tells you how well a capacitor can store charge
- Inserting a **Dielectric** between a capacitor always **increases capacitance** by a factor of \( k \)
- Capacitors store energy as well, which is given by

\[ E_{\text{capacitor}} = \frac{1}{2} QV = \frac{1}{2} CV^2 \]
- For a parallel plate capacitor with plates of area \( A \) separated by a distance \( d \), capacitance is,

\[ C = \varepsilon_0 \frac{A}{d} \]
### Combining Resistors

<table>
<thead>
<tr>
<th>Resistors in Series</th>
<th>Resistors in Parallel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{eq} = R_1 + R_2$</td>
<td>$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$</td>
</tr>
</tbody>
</table>

Note: Resistors in Series always have the same current.

If circuit has only 1 battery, choose resistors, two at a time, and reduce to a single resistor to determine the current through the battery. Then determine how current breaks up at junctions using these rules.

Resistors in Parallel always have the same voltage.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>One resistor is 3 times larger than the other, smaller resistor gets $\frac{3}{4}$ of the total current</td>
<td>$I_{R_1} = \frac{3}{4} I_{tot}$</td>
</tr>
<tr>
<td>One resistor is 5 times larger than the other, smaller resistor gets $\frac{5}{6}$ of the total current</td>
<td>$I_{R_1} = \frac{5}{6} I_{tot}$</td>
</tr>
</tbody>
</table>

or, if resistors are not a nice ratio use this formula:

$$I_{R_1} = I_{tot} \cdot \frac{R_2}{(R_1 + R_2)}$$

### Combining Capacitors

<table>
<thead>
<tr>
<th>Capacitors in Series</th>
<th>Capacitors in Parallel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$</td>
<td>$C_{eq} = C_1 + C_2$</td>
</tr>
</tbody>
</table>

Note: Capacitors in series all have the same charge.

When you reduce all capacitors to a single $C$, you can find $Q (=CV)$. Then work backwards to find $Q$ on each capacitor.

Note: Capacitors in parallel all have the same voltage.

For capacitors in parallel, if one capacitor has 3 times more capacitance than the other, it gets $\frac{3}{4}$ of the total charge.

Or, if not a nice ratio:

$$Q_1 = Q_{tot} \cdot \frac{C_1}{C_1 + C_2}$$

Note: After a short time, current will no longer flow through a $C$, and any segment of a circuit with a $C$ will have no current.
**Kirchoff’s Rules**

**Junction Rule:** \( I_{in} = I_{out} \)
- Total current flowing into junction equals total current flowing out of junction

**Loop Rule:** \( \Sigma \Delta V = 0 \)
- The sum of the changes in voltage around any closed loop always equals zero

\[ \Delta V = -IR \quad \text{(if you pass through resistor in the same direction as current)} \]
\[ \Delta V = IR \quad \text{(if you pass through resistor in the opp. direction as current)} \]

\[ \Delta V = +\varepsilon_{\text{battery}} \quad \text{(if you pass through the battery from – terminal to + terminal)} \]
\[ \Delta V = -\varepsilon_{\text{battery}} \quad \text{(if you pass through the battery from + terminal to - terminal)} \]

**Terminal Voltage**

\( V_{ab} = \varepsilon - Ir \)  
\( V_{ab} \) is the terminal voltage, \( \varepsilon \) is the emf of battery, \( r \) is internal resistance
- Every battery has an internal resistance \( r \) which will lower the terminal voltage when current flows
- A 9V battery will not necessarily have a measured terminal voltage of 9V, unless no current flows
- The \( \varepsilon \) of a 9V battery is 9V even when no current flows, but the measured terminal voltage will be less
- Slope of \( V_{ab} \) vs. I graph is negative the internal resistance. The y intercept is the emf \( \varepsilon \).

**Electrical Meters**

**Voltmeter**
- Measures voltage change across circuit element (resistor, battery, etc.)
- Ideally has infinite resistance so it does not draw any current away from circuit
- Needs to be hooked up in parallel with circuit element

![Voltmeter Diagram](image)

**Ammeter**
- Measures current through a circuit element (resistor, battery, etc.)
- Ideally has no resistance so it does not change the current
- Needs to be hooked up in series with circuit element