

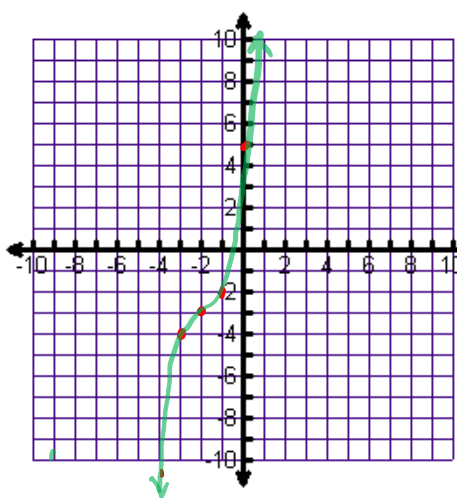
Graph each equation

1.

$$f(x) = (x + 2)^3 - 3$$

inflection pt. $(-2, -3)$

x	y
-4	-11
-3	-4
-2	-3 ✓
-1	-2
0	5



List ALL transformations

4.

$$f(x) = -2(-x + 1)^3 - 5$$

x must be positive & by itself.

$$f(x) = -2(-(x - 1))^3 - 5$$

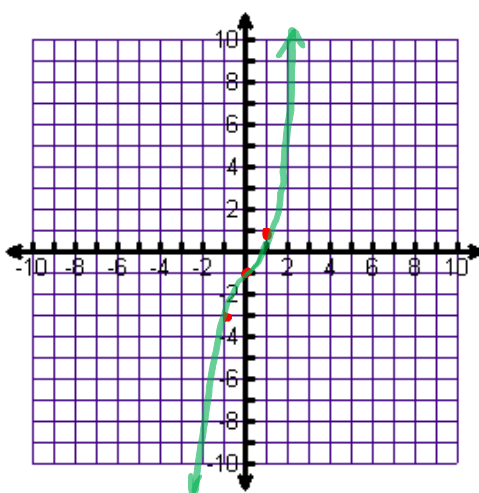
$a = -2$ reflected over x-axis
vertical stretch by 2
 $b = -1$ reflected over y-axis
 $h = 1$ shift right by 1
 $k = -5$ shift down by 5

2.

$$g(x) = 2(x)^3 - 1$$

inflection pt. $(0, -1)$

x	y
-2	-17
-1	-3
0	-1
1	1
2	15



5.

$$g(x) = \frac{1}{8}(x + 4)^3 + 9$$

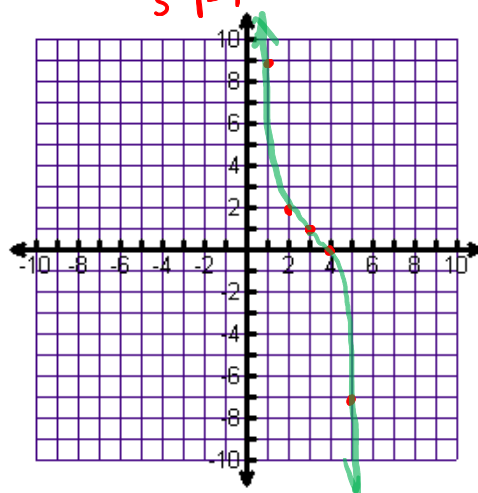
$a = \frac{1}{8}$ vertical compression by $\frac{1}{8}$
 $h = -4$ translated to the left by 4
 $k = 9$ shift up by 9

3.

$$h(x) = -27\left(\frac{1}{3}x - 1\right)^3 + 1$$

inflection pt. $(3, 1)$

x	y
1	9
2	2
3	1
4	0
5	-7



6.

$$h(x) = -\left(\frac{2}{3}x - 4\right)^3 + 2$$

we want x to be by itself and alone

$$h(x) = -\left(\frac{2}{3}(x - 6)\right)^3 + 2$$

$a = -1$ reflected over x-axis
 $b = \frac{2}{3}$ horizontal stretch by $\frac{3}{2}$
 $h = 6$ translated to the right by 6
 $k = 2$ shift up by 2

Write the inverse of each function

6.

$$f(x) = 4(x-5)^3 - 8$$

$$y = 4(x-5)^3 - 8$$

$$x = 4(y-5)^3 - 8$$

$$\frac{x+8}{4} = \frac{4(y-5)^3}{4}$$

$$\sqrt[3]{\frac{x+8}{4}} = \sqrt[3]{(y-5)^3}$$

$$\sqrt[3]{\frac{x+8}{4}} = y-5$$

$$5 + \sqrt[3]{\frac{x+8}{4}} = y$$

$$y = \sqrt[3]{\frac{x+8}{4}} + 5$$

7.

$$g(x) = -(-4(x+3))^3 + 3$$

$$y = -(-4(x+3))^3 + 3$$

$$x = -(-4(y+3))^3 + 3$$

$$\frac{x-3}{-1} = \frac{-(-4(y+3))^3}{-1}$$

$$\sqrt[3]{-(x-3)} = \sqrt[3]{-4(y+3)^3}$$

$$\sqrt[3]{-(x-3)} = \frac{-4(y+3)}{-4}$$

$$-\frac{1}{4}\sqrt[3]{-(x-3)} = y+3$$

$$-\frac{1}{4}\sqrt[3]{-(x-3)} - 3 = y$$

$$y = -\frac{1}{4}\sqrt[3]{-(x-3)} - 3$$

8.

$$h(x) = -2\sqrt[3]{x+1} - 6$$

$$y = -2\sqrt[3]{x+1} - 6$$

$$x = -2\sqrt[3]{y+1} - 6$$

$$\frac{x+6}{-2} = \frac{-2\sqrt[3]{y+1}}{-2}$$

$$\left(\frac{x+6}{-2}\right)^3 = \left(\sqrt[3]{y+1}\right)^3$$

$$\left(\frac{x+6}{-2}\right)^3 = y+1$$

$$\left(\frac{x+6}{-2}\right)^3 - 1 = y$$

$$y = \left(\frac{1}{2}(x+6)\right)^3 - 1$$

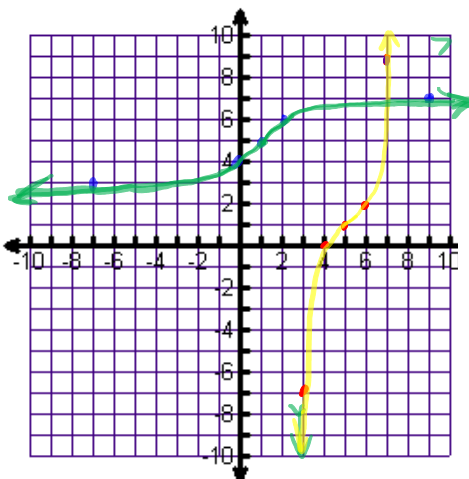
Graph the inverse of each function

9.

$$f(x) = (x-5)^3 + 1$$

inflection pt. (5, 1) Inverse

x	y		x	y
3	-7	→	-7	3
4	0	→	0	4
5	1	→	1	5
6	2	→	2	6
7	9	→	9	7

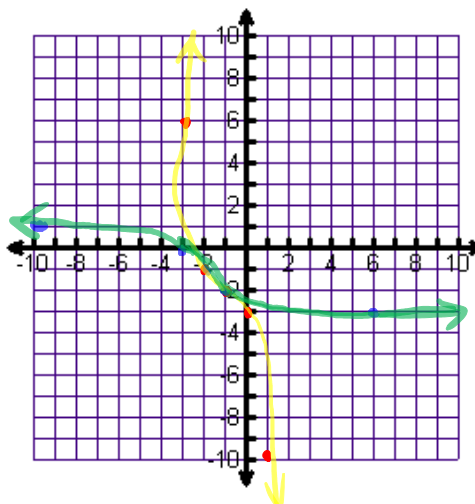


10.

$$g(x) = -(x+1)^3 - 2$$

inflection pt (-1, -2) Inverse

x	y		x	y
-3	6	→	6	-3
-2	-1	→	-1	-2
-1	-2	→	-2	-1
0	-3	→	-3	0
1	-10	→	-10	1

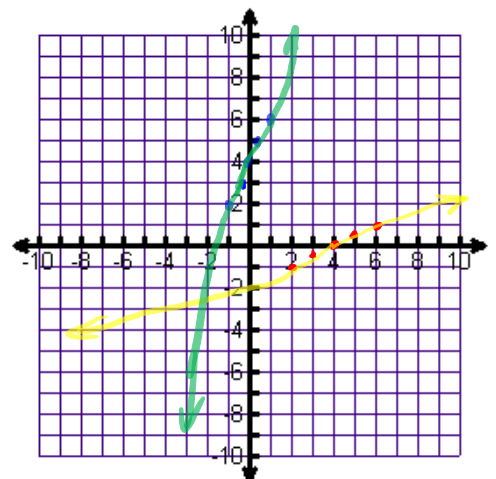


11.

$$h(x) = \left(\frac{1}{2}x - 2\right)^3 = \left(\frac{1}{2}(x-4)\right)^3$$

inflection pt. (4, 0) Inverse

x	y		x	y
2	-1	→	-1	2
3	-1/8	→	-1/8	3
4	0	→	0	4
5	1/8	→	1/8	5
6	1	→	1	6



Prove by composition

12.

$$f(x) = \frac{1}{2}x^3 - 7$$

$$g(x) = \sqrt[3]{2x + 14}$$

$$\begin{aligned} f(g(x)) &= \frac{1}{2} \left(\sqrt[3]{2x + 14} \right)^3 - 7 \\ &= \frac{1}{2} (2x + 14) - 7 \\ &= 1x + 7 - 7 \end{aligned}$$

$$\begin{aligned} f(g(x)) &= x \\ \therefore \text{inverse} \end{aligned}$$

13.

$$f(x) = 2(x - 9)^3$$

$$g(x) = \frac{1}{2} \sqrt[3]{x + 9}$$

$$f(g(x)) = 2 \left(\left(\frac{1}{2} \sqrt[3]{x + 9} \right) - 9 \right)^3$$

can't cancel or combine terms $\therefore$ not inverse

proof:

$$\text{if } x = 0 \checkmark$$

$$f(0) = 2(0 - 9)^3$$

$$f(0) = -1458$$

$$g(-1458) = \frac{1}{2} \sqrt[3]{-1458 + 9}$$

$$g(-1458) = -5.66 \checkmark$$

not inverse

14.

$$f(x) = \left(\frac{1}{2}x - 3 \right)^3$$

$$g(x) = 2\sqrt[3]{x} + 6$$

$$\begin{aligned} f(g(x)) &= \left(\frac{1}{2} (2\sqrt[3]{x} + 6) - 3 \right)^3 \\ &= \left(\sqrt[3]{x} + 3 - 3 \right)^3 \\ &= \left(\sqrt[3]{x} \right)^3 \end{aligned}$$

$$\begin{aligned} f(g(x)) &= x \\ \therefore \text{inverse} \end{aligned}$$

Solve for x and check your solution

15.

$$\frac{3\sqrt[3]{x}}{3} = \frac{12}{3}$$

$$(\sqrt[3]{x})^3 = (4)^3$$

$$\boxed{x = 64}$$

to check

$$3\sqrt[3]{64} \stackrel{?}{=} 12$$

$$3(4) \stackrel{?}{=} 12$$

$$12 = 12 \checkmark$$

16.

$$\sqrt[3]{4x} + 3 = 67$$

$$\frac{-3}{-3} \quad \frac{-3}{-3}$$

$$(\sqrt[3]{4x})^3 = (64)^3$$

$$\frac{4x}{4} = \frac{262,144}{4}$$

$$\boxed{x = 65,536}$$

to check:

$$\sqrt[3]{4(65,536)} + 3 \stackrel{?}{=} 67$$

$$\sqrt[3]{262,144} + 3 \stackrel{?}{=} 67$$

$$64 + 3 \stackrel{?}{=} 67$$

$$67 = 67 \checkmark$$

17.

$$(7x + 15)^{\frac{1}{3}} = 1$$

$$(\sqrt[3]{7x+15})^3 = (1)^3$$

$$\frac{7x+15}{-15} = \frac{1}{-15}$$

$$\frac{7x}{7} = \frac{-14}{7}$$

$$\boxed{x = -2}$$

to check

$$(7(-2) + 15)^{\frac{1}{3}} \stackrel{?}{=} 1$$

$$(-14 + 15)^{\frac{1}{3}} \stackrel{?}{=} 1$$

$$1^{\frac{1}{3}} \stackrel{?}{=} 1$$

$$1 = 1 \checkmark$$

18.

$$\sqrt[3]{(3x+8)^2} = 1$$

$$(\sqrt[3]{(3x+8)^2})^3 = (1)^3$$

$$\sqrt{(3x+8)^2} = \sqrt{1}$$

$$3x+8 = \pm 1$$

Case I

$$3x+8 = 1$$

$$\frac{-8}{-8} \quad \frac{-8}{-8}$$

$$\frac{3x}{3} = \frac{-7}{3}$$

$$\boxed{x = -\frac{7}{3}}$$

to check

$$\sqrt[3]{(3(-\frac{7}{3})+8)^2} \stackrel{?}{=} 1$$

$$\sqrt[3]{(-7+8)^2} \stackrel{?}{=} 1$$

$$\sqrt[3]{1^2} \stackrel{?}{=} 1$$

$$1 = 1$$

Case II

$$3x+8 = -1$$

$$\frac{-8}{-8} \quad \frac{-8}{-8}$$

$$\frac{3x}{3} = \frac{-9}{3}$$

$$\boxed{x = -3}$$

$$\sqrt[3]{(3(-3)+8)^2} \stackrel{?}{=} 1$$

$$\sqrt[3]{(-9+8)^2} \stackrel{?}{=} 1$$

$$\sqrt[3]{(-1)^2} \stackrel{?}{=} 1$$

$$1 = 1$$

19.

$$\sqrt[3]{x^2-5x+5} - \sqrt[3]{x} = 0$$

$$\frac{+\sqrt[3]{x}}{+\sqrt[3]{x}}$$

$$(\sqrt[3]{x^2-5x+5})^3 = (\sqrt[3]{x})^3$$

$$x^2-5x+5 = x$$

$$\frac{-x}{-x} \quad \frac{-x}{-x}$$

$$x^2-6x+5 = 0$$

$$(x-5)(x-1) = 0$$

$$\boxed{x=5} \quad x=1$$

To check:

$$\sqrt[3]{5^2-5(5)+5} - \sqrt[3]{5} \stackrel{?}{=} 0$$

$$\sqrt[3]{25-25+5} - \sqrt[3]{5} \stackrel{?}{=} 0$$

$$\sqrt[3]{5} - \sqrt[3]{5} \stackrel{?}{=} 0$$

$$0 = 0$$

$$\sqrt[3]{1^2-5(1)+5} - \sqrt[3]{1} \stackrel{?}{=} 0$$

$$\sqrt[3]{1+0} - \sqrt[3]{1} \stackrel{?}{=} 0$$

$$1 - \sqrt[3]{1} \neq 0$$

20.

$$(\sqrt[3]{2x+10})^3 = (\sqrt[3]{8x-8})^3$$

$$2x+10 = 8x-8$$

$$\frac{-10}{-10} \quad \frac{-10}{-10}$$

$$\frac{2x}{-8x} = \frac{8x-18}{-8x}$$

$$\frac{-6x}{-6} = \frac{-18}{-6}$$

$$\boxed{x = 3}$$

to check

$$\sqrt[3]{2(3)+10} \stackrel{?}{=} \sqrt[3]{8(3)-8}$$

$$\sqrt[3]{6+10} \stackrel{?}{=} \sqrt[3]{24-8}$$

$$\sqrt[3]{16} = \sqrt[3]{16} \checkmark$$