# Frame of Reference

Inertial frame of reference–
• a reference frame with a constant speed
• a reference frame that is not accelerating
• If  frame "A" has a constant speed with respect to an inertial frame "B", then frame "A" is also an inertial frame of reference.

Newton's 3 laws of motion are valid in an inertial frame of reference.

Example:
We consider the earth or the "ground" as an inertial frame of reference. Observing from an object in motion with a constant speed (a = 0) is an inertial frame of reference

A **non-inertial frame of reference** is one that is accelerating and Newton's laws of motion appear invalid unless fictitious forces are used to describe the motion of objects observed in the non-inertial reference frame.

Example:
If you are in an automobile when the brakes are abruptly applied, then you will feel pushed toward the front of the car. You may actually have to extend you arms to prevent yourself from going forward toward the dashboard. However, there is really no force pushing you forward. The car, since it is slowing down, is an accelerating, or non-inertial, frame of reference, and the law of inertia (Newton's $1^{st}$ law) no longer holds if we use this non-inertial frame to judge your motion. An observer outside the car however, standing on the sidewalk, will easily understand that your motion is due to your inertia…you continue your path of motion until an opposing force (contact with the dashboard) stops you. No "fake force" is needed to explain why you continue to move forward as the car stops.

Many of our problems will be in the inertial frame of reference where we analyze the motion of an object (or objects) from the point of view of the ground (which we consider motionless). Sometimes we will observe from a  moving reference point that may or may not be at constant speed.  Either way, **this will change the way the motion of an object is perceived or measured compared to a stationary reference frame.**

## Common examples of frames of reference

### Inertial

An elevator stopped at the 2nd floor
A car moving on a linear path at constant speed
The bank of a river
A river current that does not accelerate

### Non-inertial

An elevator that starts moving up after the button for the 4th floor is pressed
A car that is speeding up to get through a yellow light
An object in circular motion at constant speed (why?)
An object in circular motion with varying speed

If an object is observed in the non-inertial frames of reference above, can you describe how the law of inertia appears to be violated in each example?

# Descriptions of relative motion

Example 1: if you are standing on a sidewalk and observe a person in a car drive by at 30 km/hr you will say that the driver is moving toward you or away from you at 30 km/hr.

But what if you are observing the motion of the driver from within the car? In this case, you would say that the driver is not moving, even though the car is moving at a constant speed of 30 km/hr relative to a person standing on the sidewalk. In fact, you would say the person on the sidewalk is moving toward or away from you at 30 km/hr!

The two frames of reference are the ground and the car.

Example 2: You are floating down a river on a raft at speed of 4 m/s. From the shore, your friend clocks your speed at 4 m/s. You look down at the river and see a stick floating in the water. You yell to your friend that the stick is not moving. Your friend, yells back, "no, it is moving at 4 m/s, just like you." You yell back, "no, you are moving backward at 4 m/s."

The two frames of reference are the riverbank and the river (or the raft).

**How to write a relative motion equation**

An object's velocity is relative to the observer's position or reference frame.

If the velocity of object A is known in reference frame B and the velocity of B is known in reference frame C, then the velocity of A can be found by the following vector sum:

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

This equation reads: the velocity of A with respect to C is equal to the velocity of A with respect to B plus the velocity of B with respect to C. Reference frame B is the intermediate reference frame. If I switch the order of the subscripts, I reverse the direction. For example: $+ v_{AC} = - v_{CA}$ the signs indicate direction; if A with respect to C is due East, then C with respect to A is due West.

Let's say A is a boat moving on a river B and C is the riverbank. The boat is traveling 30 m/s on a river with a current (B) of 5 m/s. Since both the boat and current are going in the same direction, the vector sum is 35 m/s and that is how fast the boat appears to be going to an observer on the bank (C). To extend this concepta little, if you were on a raft moving with the current, the boat would appear to you to be moving at 25 m/s. Can you see why? Can you write a vector equation that tells you the velocity of the boat relative to the raft, using the riverbank as an intermediate reference frame?

## Vector Analysis, 1D

You're walking along a road, heading west at 8 km/hr. A car is going by with velocity of 30 km/hr east. There is also a truck passing, traveling at 40 km/hr west.

How fast is the truck traveling relative to you?
you solve

How fast is the car traveling relative to you?
you solve

How fast is the car traveling relative to the truck?
solution:

$v_{CT} = v_{CG} + v_{GT}$ velocity of the car relative to the truck = velocity of the car relative to the ground + the velocity of the ground relative to the truck.

Since these vectors only have an "x" component, we simply add the magnitudes to solve (watch the algebraic sign…must get that right!)

let East be (+) and West be (-)
$v_{CG} = + 30$ km/hr
$v_{TG} = - 40$ km/hr BUT, $v_{GT} = + 40$ km/hr

therefore $v_{CT} = v_{CG} + v_{GT} = + 30$ km/hr + (+40 km/hr)

$v_{CT} = +70$ km/hr or 70km/hr East relative to the truck

## Vector Analysis, 2D

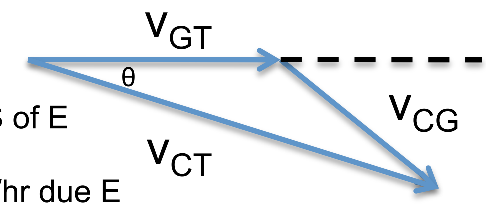
Let's change the 1-D example to 2-D. The truck still moves at 40 km/hr west, but the car turns on to a road going 40 degrees south of east, and travels at 30 km/hr. What is the velocity of the car relative to the truck now?

The relative velocity equation for this situation still looks like the 1D eq but you have to resolve the position vectors to solve it.

$v_{CT} = v_{CG} + v_{GT}$

$v_{CG} = 30$ km/hr @40 deg S of E

$v_{GT} = + 40$ km/hr or 40 km/hr due E




$v_{GT} = [(V_{CG,x} + V_{GT,x})^2 + (V_{GC,y} + V_{GT,y})^2]^{1/2}$

$\theta = \tan^{-1} (V_{CG,y} + V_{GT,y})/(V_{CG,x} + V_{GT,x})$

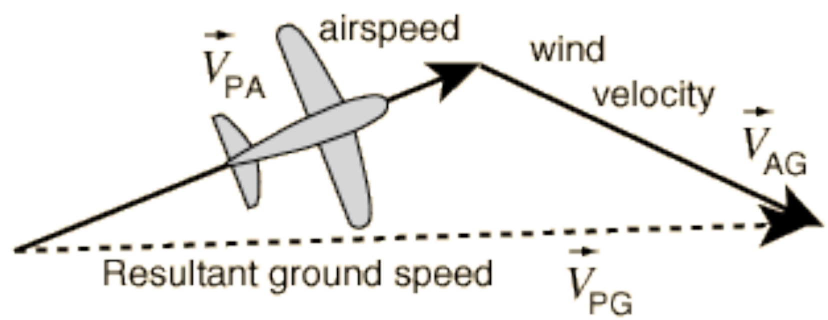
$V_{CG,x} = 30\cos 40$ $V_{GT,x} = 40$

$V_{CG,y} = 30\sin 40$ $V_{GT,y} = 0$

Plug and play…

$v_{CT} =$ 66 km/hr @ 17 deg S of E

# Vector Analysis, another 2D example




If the plane above is traveling with an airspeed of $V_{PA}$ = 900 km/hr and encounters a wind speed of $V_{AG}$ = 50 km/hr then what is the ground speed?

The ground speed is just the **vector sum** of the airspeed and the wind speed. Find the x and y components of the airspeed and wind speed and use them to find the resultant ground speed.

$$\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG}$$

See if you can solve this using the 2D car and truck problem on the previous page.

Vector Analysis: Position relative to a moving object

Two cars approach a crossroad 36 m ahead. Car *A* is traveling east at 9 m/s and car *B* is traveling south at 12 m/s.

**a.** What is the location of car *B* relative to car *A*?

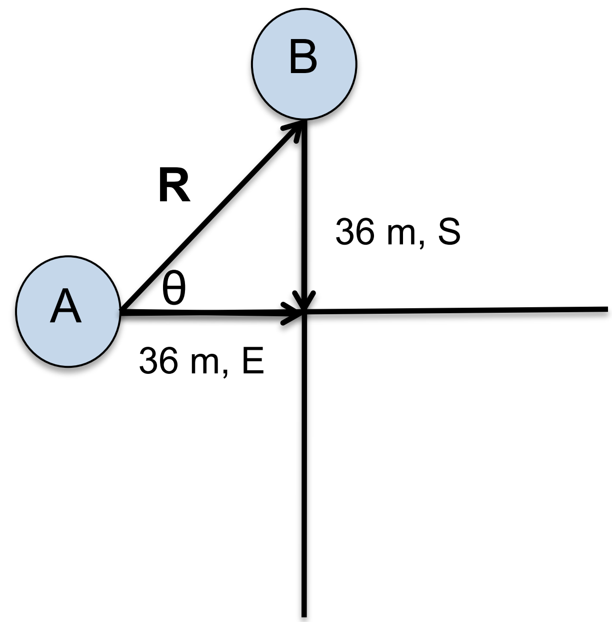
$$R = \sqrt{(36\ \text{m})^2 + (36\ \text{m})^2}$$

$$R = 51\ \text{m}$$

$$\tan\theta = \frac{B}{A} = \frac{36\ \text{m}}{36\ \text{m}} = 1.0$$

$$\theta = \tan^{-1}(1.0) = 45°$$

$R_{\text{car } B \text{ relative to car } A}$ = **51 m at 45° north of east**




In this case, we need to find the position vector between car A and B. Since they happen to be the same distance from a reference point (the crossroad), it is sort of easy to see that the angle of B relative to A will be 45 deg (the x and y components of the position vector make a 45-45-90 triangle!)